SOLUTIONS MANUAL TO ACCOMPANY

MODERN POWER SYSTEM ANALYSIS

3rd Edition

D P Kothari

Professor, Centre of Energy Studies
Deputy Director (Admin.)
Indian Institute of Technology
Delhi

I J Nagrath

Adjunct Professor, and Former Deputy Director,
Birla Institute of Technology and Science
Pilani

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SOLUTIONS
Chapter 2

2.1

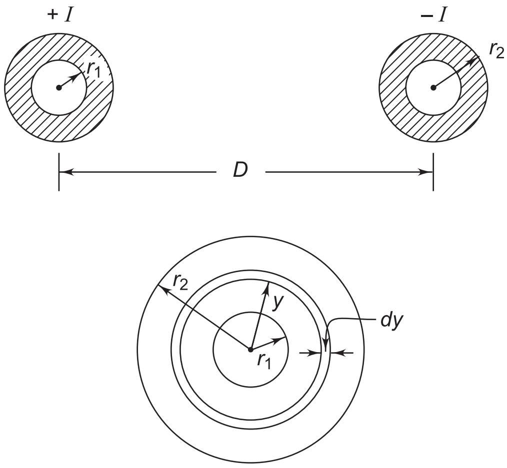


Fig. S-2.1

Assume uniform current density

$$2\pi\, y\, H_y = I_y$$

$$I_y = \left(\frac{y^2 - r_1^2}{r_1^2 - r_2^2}\right) I$$

$$\therefore \qquad H_y = \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right) \times \frac{1}{2\pi y}\, I$$

$$d\phi = \mu\, H_y\, d_y$$

$$d\lambda = \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right) I d\phi$$

$$= \mu \left(\frac{y^2 - r_1^2}{r_2^2 - r_1^2}\right)^2 \frac{I}{2\pi y}\, dy$$

$$= \frac{\mu I}{2\pi} \times \frac{y^3 - 2\, r_1^2\, y + r_1^4 / y}{\left(r_2^2 - r_1^2\right)^2}\, dy$$

Integrating

$$\lambda_{\text{int}} = \frac{\mu I}{2\pi\,(r_2^2 - r_1^2)^2} \int_{r_1}^{r_2} \left[y^3 - 2 r_1^2 y + r_1^4 / y\right] dy$$

$$= \frac{\mu I}{2\pi\,(r_2^2 - r_1^2)^2} \left\{ \frac{y^4}{4}\Bigg|_{r_1}^{r_2} - r_1^2 y^2 \Big|_{r_1}^{r_2} + r_1^4 \ln y \Big|_{r_1}^{r_2} \right\}$$

$$= \frac{\mu I}{2\pi (r_2^2 - r_1^2)^2}\left\{\frac{1}{4}(r_2^4 - r_1^4) - r_1^2(r_2^2 - r_1^2) + r_1^4 \ln \frac{r_2}{r_1}\right\}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\qquad \mu_r = 1$

$$L_{int} = \frac{\frac{1}{2}\times 10^{-7}}{\left(r_2^2 - r_1^2\right)^2}\left[\left(r_2^4 - r_1^4) - 4r_1^2\left(r_2^2 - r_1^2\right) + 4r_1^4 \ln \frac{r_2}{r_1}\right)\right]$$

$$L_{ext}\ (1) = 2 \times 10^{-7} \ln \frac{D}{r_2} = L_{ext}\ (2);\ \text{assuming } D >> r_2$$

Line inductance = 2 (L_{int} + L_{ext} (1)) H/m.

2.2.

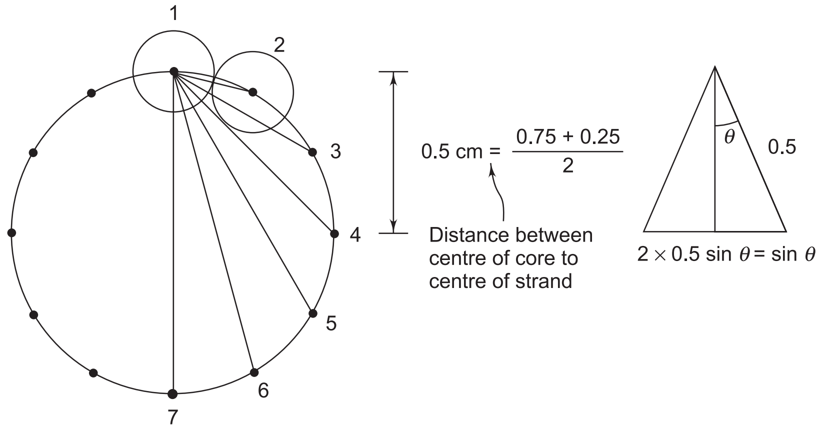


Fig. S-2.2

Diameter of nonconducting core = 1.25 – 2 × (0.25) = 0.75 cm

Note: Core is nonconducting.

D_{12} = sin 15º = 0.259 cm $\qquad D_{13}$ = sin 30º = 0.5 cm

D_{14} = sin 45º = 0.707 cm $\qquad D_{15}$ = sin 60º = 0.866 cm

D_{16} = sin 75º = 0.965 cm $\qquad D_{17}$ = sin 90º = 1.0 cm

$D_{11} = r'$ = (0.25/2) × 0.7788 = 0.097 cm

$$D_s = \{(0.097 \times 1) \times (0.259)^2 \times (0.5)^2 \times (0.707)^2 \times (0.866)^2 \times (0.965)^2\}^{1/12}$$

= 0.536 cm

$D_m \approx 1$ m

$$L = 2 \times 0.461 \log \frac{100}{0.536} = 2.094 \text{ mH/km}$$

$X = 314 \times 2.094 \times 10^{-3} =$ **0.658 Ω/km**

2.3 $H_y = I/2\pi y$

$$d\phi = \frac{\mu I}{2\pi y} dy$$

$$d\lambda = 1 \times d\phi = \frac{\mu I}{2\pi y} dy$$

$$\lambda = \frac{\mu}{2\pi} I \int_r^R \frac{dy}{y} = \mu \frac{I}{2\pi} \ln \frac{R}{r}$$

$$L = \frac{\mu}{2\pi} \ln \frac{R}{r} \text{ H/m}$$

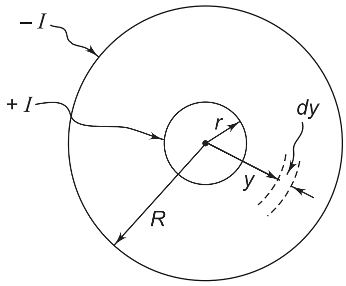


Fig. S-2.3

2.4 Flux linkage of sheath loop due to cable current = $2 \times 2 \times 10^{-7} \times 800 \times \ln \frac{0.5 \times 200}{7.5}$ Wb-T/m

$$\text{Voltage induced in sheath} = 314 \times 0.32 \ln \frac{100}{7.5} \text{ V/km}$$

$$= \mathbf{260.3\ V/km}$$

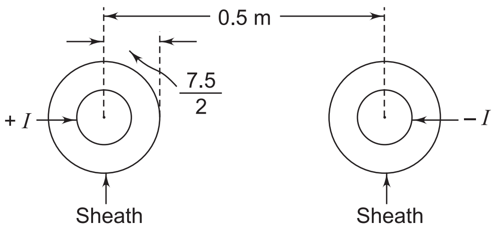


Fig. S-2.4

2.5 $H_P = \frac{I}{2\pi \times 3d} - \frac{I}{2\pi\, d} = \frac{I}{2\pi d}\left(\frac{1}{3} - 1\right) = -\frac{I}{3\pi d}$ AT/m^2

(direction upwards)

2.6

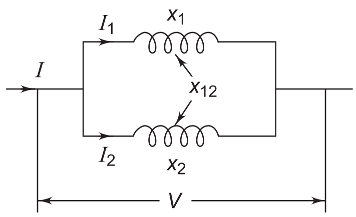


Fig. S-2.6

$$V = j\, X_1\, I_1 + j\, X_{12}\, I_2 = j\, X_2\, I_2 + j\, X_{12}\, I_1$$

$$I = I_1 + I_2\,; \quad I_1 = \frac{V}{j\,(X_1 - X_{12})}; \; I_2 = \frac{V}{j\,(X_2 - X_{12})}$$

$$I = \frac{V}{j}\left[\frac{1}{X_1 - X_{12}} + \frac{1}{X_2 - X_{12}}\right] = \frac{V}{j\,X}$$

$$\therefore \qquad X = \frac{(X_1 - X_{12})(X_2 - X_{12})}{X_1 + X_2 - 2X_{12}}$$

2.7

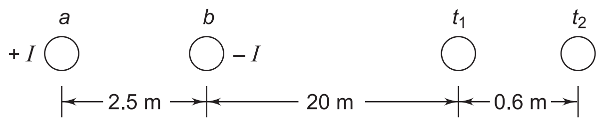


Fig. S-2.7

$$\lambda_{t1} = 2 \times 10^{-7}\left(I \ln\frac{1}{22.5} - I \ln\frac{1}{20}\right)$$

$$= 2 \times 10^{-7} \times 150 \ln\frac{20}{22.5}$$

$$= -0.353 \times 10^{-5} \text{ Wb-T/m}$$

$$\lambda_{t2} = 2 \times 10^{-7} \times 150 \left(\ln\frac{1}{23.1} - \ln\frac{1}{20.6}\right)$$

$$= -0.343 \times 10^{-5} \text{ Wb-T/m}$$

$$\lambda_t = \lambda_{t1} - \lambda_{t2} = -0.01 \times 10^{-5} \text{ Wb-T/m}$$

Mutual inductance $= (0.01 \times 10^{-5}/150) \times 10^3 \times 10^3$ mH/km

$= 0.00067$ mH/km

Induced voltage in telephone line $= 314 \times 0.01 \times 10^{-5} \times 10^3$

$=$ **0.0314 V/km**

2.8 $I_a = 400 \angle 0^\circ$, $I_b = 400\angle -120^\circ$, $I_c = 400\angle 120^\circ$

Using Eq. (2.40)

$$\lambda_t = 2 \times 10^{-7} \times 400 \left(\ln\frac{26}{25} + 1\angle -120^\circ \times \ln\frac{21}{20} + 1\angle 120^\circ \ln\frac{16}{15}\right) \text{ Wb-T/m}$$

$$= 0.0176 \times 10^{-4} \angle 140^\circ \text{ Wb-T/m}$$

$$\text{Mutual inductance} = \frac{0.0176 \times 10^{-4} \angle 140^\circ}{400} \times 10^6$$

$$= \frac{1.76}{400}\angle 140^\circ \text{ mH/km}$$

$= \mathbf{0.0044\angle 140^\circ}$ **mH/km**

Voltage induced in telephone line $= 314 \times 0.0176 \times 10^{-4} \times 10^3 \angle 140^\circ$

$= \mathbf{0.553\angle 140^\circ}$ **V/km**

2.9 Here $d = 15$ m, $s = 0.5$ m

Using method of GMD

$$D_{ab} = D_{bc} = [d\ (d + s)\ (d - s)d]^{1/4}$$
$$= (15 \times 15.5 \times 14.5 \times 15)^{1/4} = 15 \text{ m}$$
$$D_{ca} = [2d\ (2d + s)\ (2d - s)\ 2d]^{1/4}$$
$$= (30 \times 30.5 \times 29.5 \times 30)^{1/4} = 30 \text{ m}$$
$$D_{eq} = (15 \times 15 \times 30)^{1/3} = 18.89 \text{ m}$$
$$D_s = (r'\ sr'\ s)^{1/4} = (r'\ s)^{1/2}$$
$$= (0.7788 \times 0.015 \times 0.5)^{1/2}$$
$$= 0.0764 \text{ m}$$

Inductive reactance/phase

$$X_L = 314 \times 0.461 \times 10^{-3} \log \frac{18.89}{0.0764}$$
$$= \mathbf{0.346\ \Omega/km}$$

2.10 $$X_L = 314 \times 0.921 \times 10^{-3} \log \frac{D}{0.01} = 31.4/50$$

$\therefore$ D = **1.48** m (maximum permissible)

2.11

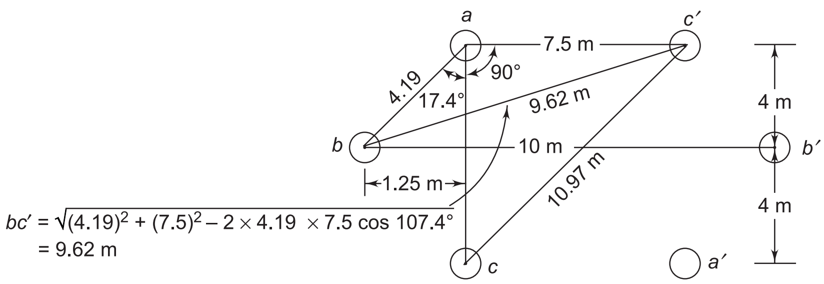


Fig. S-2.11

In section 1 of transposition cycle

$$D_{ab} = \sqrt{1.19 \times 9.62} = 6.35;\ D_{bc} = \sqrt{4.19 \times 9.62} = 6.35$$
$$D_{ca} = \sqrt{7.5 \times 8} = 7.746$$
$$D_{eq} = \sqrt[3]{6.35 \times 6.35 \times 7.746} = 6.78$$
$$D_{sa} = \sqrt{0.01 \times 10.97} = 0.3312 = D_{sc}$$
$$D_{sb} = \sqrt{0.01 \times 10} = 0.3162$$
$$D_s = \sqrt[3]{0.3312 \times 0.3312 \times 0.3162} = 0.326 \text{ m}$$
$$X = 0.314 \times 0.461 \log \frac{6.78}{0.326} = \mathbf{0.191}\ \Omega\text{/km/phase}$$

2.12 $$r' = 0.7788 \times 1.5 \times 10^{-2} = 0.0117 \text{ m}$$

$$D_{ab} = \sqrt[4]{1 \times 4 \times 1 \times 2}\ ;\ D_{bc} = \sqrt[4]{1 \times 4 \times 1 \times 2}\ ;\ D_{ca} = \sqrt[4]{2 \times 1 \times 2 \times 5}$$

$$D_m = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = \sqrt[12]{1280} = 1.815 \text{ m}$$

$$D_{sa} = D_{sb} = D_{sc} = \sqrt{0.0117 \times 3} = 0.187$$

$\therefore$ $$D_s = 0.187 \text{ m}$$

$$L = 0.461 \log \frac{1.815}{0.187} = \mathbf{0.455} \text{ mH/km/phase}$$

2.13

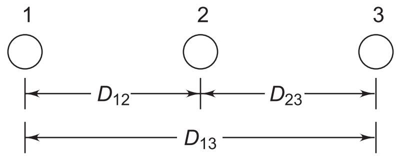


Fig. S-2.13

$$D_{13} = 2D_{12} = 2D_{23} = 2d$$

$$\sqrt[3]{2d \times d \times d} = 3$$

$$\sqrt[3]{2}\ d = 3 \qquad \therefore \quad d = \mathbf{2.38 \text{ m}}$$

2.14 Refer to Fig. 2.16 of the text book.

Case (i) $2\pi r^2 = A$

$$r = (A/2\pi)^{1/2} \quad \therefore \quad r' = 0.7788\ (A/2\pi)^{1/2}$$

$$\text{Self G.M.D} = \sqrt{r'd} = \sqrt{(0.7788)\, d\, (A/2\pi)^{1/2}}$$

$$= \mathbf{0.557\ d^{1/2}\ A^{1/4}}$$

Case (ii) $3\pi r^2 = A \quad \therefore \quad r = \sqrt{A/3\pi}$

$$\text{Self GMD} = (r'dd)^{1/3} = \sqrt{(0.7788)^{1/3}\, (A/3\pi)^{1/6}\, d^{2/3}}$$

$$= \mathbf{0.633\ d^{2/3}\ A^{1/6}}$$

Case (iii) $4\pi r^2 = A \quad \therefore \quad r = \sqrt{A/4\pi}$

$$\text{Self GMD} = \sqrt[4]{r'\, dd\, 2^{1/2} d}$$

$$= 1.09\ \sqrt[4]{r'\, d^3}$$

$$= 1.09\ (0.7788)^{1/4} \left(\frac{A}{4\pi}\right)^{1/8} d^{3/4}$$

$$= \mathbf{0.746\ d^{3/4}\ A^{1/8}}$$

Chapter 3

3.1
$$V_a = \frac{1}{\sqrt{3}}|V| \angle 0^\circ$$

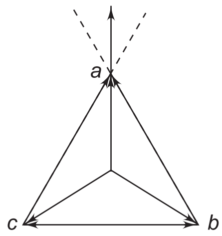


Fig. S-3.1

$$V_{ab} = |V| \angle 30^\circ$$
$$V_{bc} = |V| \angle -90^\circ$$
$$V_{ca} = |V| \angle 150^\circ$$
$$D_{ab} = D_{bc} = D$$
$$D_{ac} = 2D$$
$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{D}{2D}\right)$$
$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln\frac{2D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{2D}\right)$$
$$V_{ab} = \frac{1}{2\pi k}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{1}{2}\right) = |V| \angle 30^\circ \quad \text{(i)}$$
$$V_{ac} = \frac{1}{2\pi k}\left(q_a \ln\frac{2D}{r} + q_c \ln\frac{r}{2D}\right) = |V| \angle -30^\circ \quad \text{(ii)}$$
$$q_a + q_b + q_c = 0 \quad \text{(iii)}$$

Eliminating q_b from (i) with the help of (iii)

$$2q_a \ln\frac{D}{r} + q_c \ln\frac{D}{2r} = 2\pi k\,|V| \angle 30^\circ \quad \text{(iv)}$$

Eliminating q_c between (ii) and (iv)

$$2q_a \ln\frac{D}{r} \ln\frac{r}{2D} - q_a \frac{2D}{r} \ln\frac{D}{2r} = 2\pi k\,|V| \left[\ln\frac{r}{2D}\angle 30^\circ - \ln\frac{D}{2r}\angle -30^\circ\right]$$

$$\therefore \quad q_a = \frac{2\pi k\,|V|\left[\ln\frac{r}{2D}\angle 30^\circ - \ln\frac{D}{2r}\angle -30^\circ\right]}{2\ln\frac{D}{r}\ln\frac{r}{2D} - \ln\frac{2D}{r}\ln\frac{D}{2r}} \text{ F/m} \quad \text{(v)}$$

$$I_a = 2\pi f\, q_a \angle 90^\circ \text{ A} \qquad \text{(with } q_a \text{ given in v)} \quad \text{(vi)}$$

3.2 Mutual GMD (calculated from the first transposition cycle)

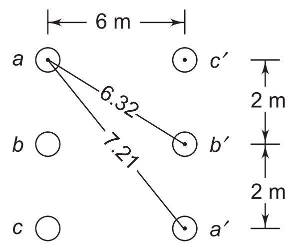


Fig. S-3.2

$$r = 0.01 \text{ m}$$
$$D_{ab} = \sqrt{2 \times 6.32} = 3.555 = D_{bc}$$
$$D_{ca} = \sqrt{4 \times 6} = 4.899$$
$$D_{eq} = \sqrt[3]{D_{ab}D_{bc}D_{ca}} = 3.955 \text{ m}$$

Self GMD (calculated from the first transposition cycle)

$$D_{sa} = \sqrt{0.01 \times 7.21} = 0.2685 = D_{sc}$$

$$D_{sb} = \sqrt{0.01 \times 6.00} = 0.2449; \; D_s = \sqrt[3]{(0.2685)^2 \times 0.2449} = 0.261$$

$$C_n = \frac{0.0242}{\log \dfrac{3.955}{0.261}} = \mathbf{0.0204} \; \mu\mathbf{F/km}$$

3.3 $\dfrac{0.0242}{\log(4/r)} = 0.01 \; \mu\text{F/km}$

$$\log(4/r) = 2.42; \; r = \frac{4}{\log^{-1} 2.42} = 0.015 \text{ m}$$

In new configuration, $D_{eq} = \sqrt[3]{4 \times 4 \times 8} = 5.04$

$$C = \frac{0.0242}{\log \dfrac{5.04}{0.015}} = \mathbf{0.0096} \; \mu\text{F/km.}$$

3.4 Here $d = 15$ m, $s = 0.5$ m, $r = 0.015$ m

$$D_{eq} = \sqrt[3]{15 \times 15 \times 30} = 18.89$$

$$D_s = \sqrt{0.015 \times 0.5} = 0.0866$$

$$C = \frac{0.0242}{\log \dfrac{18.89}{0.0866}} = \mathbf{0.0103} \; \mu\text{F/km to neutral}$$

3.5

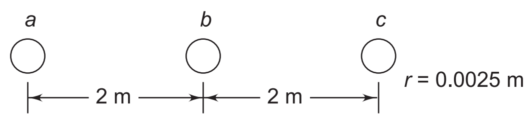


Fig. S-3.5

At a certain instant $q_a = q_b = q$

$\because \quad q_a + q_b + q_c = 0 \quad \therefore \quad q_c = -2q$

$$V_{ab} = \frac{1}{2\pi k}\left(q \ln \frac{2}{0.0025} + q \ln \frac{0.0025}{2} - 2q \ln \frac{2}{4}\right) = 775$$

$$q = -\frac{775 \times \pi k}{\ln 1/2} = -\frac{775 \times \pi \times 8.85 \times 10^{-12}}{\ln(1/2)} \times 1000$$

$$= \mathbf{3.08 \times 10^{-5}} \text{ coulomb/km}$$

3.6 $D = 7$ m $\qquad r = 0.0138$ m

$$D_{ab} = \sqrt[4]{7 \times 28 \times 7 \times 14} = 11.772; \qquad D_{bc} = 11.772$$

$$D_{ca} = \sqrt[4]{14 \times 7 \times 14 \times 35} = 14.803; \quad D_{eq} = \sqrt[3]{(11.772)^2 \times 14.803}$$
$$= 12.706$$

$$D_{sa} = \sqrt{0.0138 \times 21} = 0.538 = D_{sb} = D_{sc} \quad \therefore \quad D_s = 0.538$$

$$C = \frac{0.0242}{\log \dfrac{12.706}{0.538}} = 0.0176 \ \mu\text{F/km}$$

Susceptance $B = 314 \times 0.0176 \times 10^{-6} = \mathbf{5.53 \times 10^{-6}}$ ℧/km

3.7 $\varepsilon = \dfrac{q}{2\pi k y}$ V/m

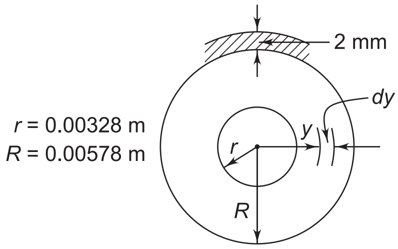


Fig. S-3.7

$$V_{12} = \int_r^R \frac{q}{2\pi k y}\, dy$$

$$V_{12} = \frac{q}{2\pi k} \ln \frac{R}{r}$$

$$C = \frac{q}{V_{12}} = \frac{2\pi k}{\ln R/r} = \frac{2\pi \times 3.8 \times 8.85 \times 10^{-12}}{\ln \dfrac{0.00578}{0.00328}}$$

$$= 373 \times 10^{-12} \text{ F/m}$$

$$X_c = \frac{1}{\omega C} = \frac{10^{12}}{314 \times 373 \times 1000} = \mathbf{8.54 \times 10^3} \ \Omega/\text{km}$$

3.8 $r = 0.01$ m

$$D_{eq} = \sqrt[3]{5 \times 6 \times 7} = 5.943$$

$$C = \frac{0.0242}{\log \dfrac{5.943}{0.01}} = \mathbf{8.72 \times 10^{-3}} \ \mu\text{F/km}$$

3.9

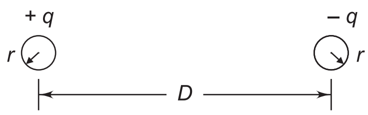


Fig. S-3.9

The expression for capacitance is derived in Sec. 3.4 [see Eq. (3.4 c)].

$$r = 0.003 \text{ m}$$
$$D = 0.35 \text{ m}$$

Electric stress is maximum at conductor surface.

$$E_{\max} = \frac{q}{2\pi k r}$$

$$q_{max} = 25 \times 10^5 \times 2\pi \times 8.85 \times 10^{-12} \times 0.003$$

$$= 150 \times \pi \times 8.85 \times 10^{-10} \text{ coulombs/m}$$

$$C_{ab} = \frac{0.0121}{\log \dfrac{0.35}{0.003}} = 5.854 \times 10^{-3} \ \mu\text{F/km}$$

$$V_{ab} \text{ (max)} = \frac{q_{max}}{C_{ab}} = \frac{150 \times \pi \times 8.85 \times 10^{-10}}{5.854 \times 10^{-3} \times 10^{-6} \times 10^{-3}}$$

$$= \mathbf{71.24} \text{ kV}$$

Chapter 4

4.1 Choose Base: 100 MVA

11 kV in generator circuit

220 kV transmission line

66 kV load bus

Reactance $T_1 = 0.1$ pu

Reactance $T_2 = 0.08$ pu

Reactance transmission line $= \dfrac{150 \times 100}{(220)^2}$

$= 0.31$ pu

Load: $\dfrac{60}{100} = 0.6$ pu MW; 0.9 pf lagging

Voltage $V_2 = \dfrac{60}{66} = 0.909\ \angle 0°$

Current $I_2 = \dfrac{0.6}{1 \times 0.9}\ \angle -25.8° = 0.6667 \angle -25.8°$ pu

Generator terminal voltage

$$V_1 = V_2 + j\,(0.1 + 0.08 + 0.31) \times 0.6667\ \angle -25.8°$$

$$= 0.909 + 0.327\ \angle 64.2°$$

$$= 1.09\ \angle 15.6°$$

$|V_1|$ (line) $= 1.09 \times 11 =$ **12** kV

4.2

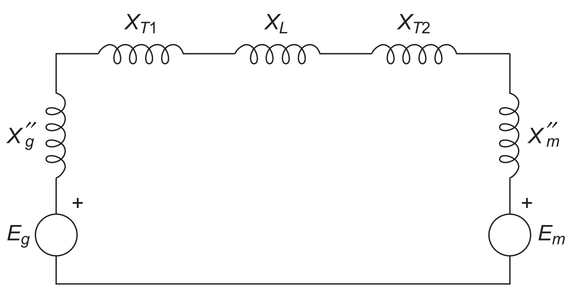


Fig. S-4.2

Base: 100 MVA

220 kV in line

$220 \times \dfrac{33}{220} = 33$ kV in generator

$$220 \times \frac{11}{220} = 11 \text{ kV in motor}$$

Per unit reactances are:

$$X''_g = 0.2 \left(\frac{100}{40}\right) \times \left(\frac{25}{33}\right)^2 = 0.287$$

$$X''_m = 0.3 \times \left(\frac{100}{50}\right) = 0.6$$

$$X_{T1} = 0.15 \times \frac{100}{40} = 0.375$$

$$X_{T2} = 0.15 \times \left(\frac{100}{30}\right) = 0.5$$

$$X_L = \frac{50 \times 100}{(220)^2} = 0.103$$

Chapter 5

5.1 $|V_R| = 11/\sqrt{3} = 6.351$ kV

(a) $$\phi_R = \cos^{-1} 0.707 = 45°;\ \ \theta = \tan^{-1}\frac{12}{10} = 50.2°$$

$$|Z| = \sqrt{10^2 + 12^2} = 15.62$$

Using Eq. (5.10)

$$|I| = \frac{2\,|V_R|}{|Z|}\sin(\phi_R + \theta - 90°)$$

$$= \frac{2 \times 6.351}{15.62}\sin 5.2° = 73.7 \text{ A}$$

$$P = \sqrt{3} \times 11 \times 73.7 \times 0.707 = \mathbf{992.75} \text{ kW}$$

(b) $$\phi_R = \cos^{-1} 0.85 = 31.8°$$

$$\phi_R + \theta - 90° = 31.8° + 50.2° - 90° = -8°$$

Since it is negative, no solution for P is possible which would give zero voltage regulation.

5.2

$a = 1$ $\quad$ $A = 0.9 \angle 1.5°$

$b = Z_T = 100 \angle 67°$ $\quad$ $B = 150 \angle 65°$

$c = 0$ $\quad$ $C = ?$

$d = 1$ $\quad$ $D = 0.9 \angle 1.5°$

$$AD - BC = 1 \quad \text{(i)} \quad \therefore C = \frac{AD-1}{B} = \frac{0.81\angle 3° - 1}{150\angle 65°} = 0.001 \angle 102.6°$$

$A' = Aa + Bc$ $\quad$ $B' = Ab + Bd$

$C' = Ca + Dc$ $\quad$ $D' = Cb + Dd$

$\therefore$ $A' = 0.9 \angle 1.5° \times 1 = \mathbf{0.9 \angle 1.5°}$

$B' = 0.9 \angle 1.5° \times 100 \angle 67° + 150 \angle 65° \times 1 = \mathbf{239.9 \angle 66.3°}$

$C' = 0.001\angle 102.6° \times 1 + 0.9\angle 1.5° \times 0 = \mathbf{0.001\angle 102.6°}$

D' can be calculated using relation (i)

$D' = (1 + B'C')/A' = \mathbf{0.85 \angle 1.96°}$

5.3 (a) $$L = 0.461 \log \frac{\sqrt[3]{4 \times 5 \times 6}}{0.7788 \times 10^{-2}} = 1.29 \text{ mH/km}$$

$$C = \frac{0.0242}{\log \dfrac{\sqrt[3]{4 \times 5 \times 6}}{10^{-2}}} = 0.009\ \mu\text{F/km}$$

$R = 200 \times 0.16 = 32\ \Omega$; $X = 314 \times 1.29 \times 10^{-3} \times 200 = 81\ \Omega$

$Z = 32 + j\ 81 = 87.1\ \angle 68.4^o$

$Y = j\ 314 \times 0.009 \times 10^{-6} \times 200 = 0.00056\ \angle 90^o$

$A = 1 + YZ/2 = 1 + 0.024\ \angle 158.4^o = \mathbf{0.978\ \angle 0.5^o} = D$

$$B = \sqrt{\frac{Z}{Y}} = YZ\left(1+\frac{yz}{6}\right) = Z\ (1 + YZ/6) = \mathbf{86.4\ \angle 68.6^o}$$

$$C = \sqrt{\frac{Y}{Z}}\ \sqrt{YZ}\,(1 + YZ/6) = Y(1 + YZ/6) = \mathbf{0.00056\ \angle 90.2^o}$$

(b) $I_R = \dfrac{50}{\sqrt{3}\times 132\times 0.8}\angle{-36.9^o} = 0.2734\ \angle{-36.9^o}$ kA

$V_R = 132/\sqrt{3}\angle 0^o$ kV $= 76.21\ \angle 0^o$ kV

$V_S = AV_R + BI_R$

$= 0.978\ \angle 0.5^o \times 76.21\ \angle 0^o + 86.4\ \angle 68.6^o \times 0.2734\ \angle{-36.9^o}$

$= 95.52\ \angle 7.8^o$ kV

$|V_S|$ (line) $= \sqrt{3}\times 95.52 =$ **165.44** kV

$I_S = CV_R + DI_R$

$= 0.00056\ \angle 90.2^o \times 76.21\ \angle 0^o + 0.978\ \angle 0.5^o \times 0.2734\angle{-36.9^o}$

$= \mathbf{0.244\ \angle{-28.3^o}}$ kA

Sending-end power factor = cos $(28.3^o + 7.8^o)$ = **0.808** lagging

Sending-end power = $\sqrt{3} \times 165.44 \times 0.224 \times 0.808$ = **56.49** MW

(c) Efficiency of transmission = 50 × 100/56.49 = 88.5%

$|V_R|$ (no load) = 165.44/0.978 = 169.16 kV

(d) Per cent regulation = (169.16 – 132) × 100/132 = **28.15%**

Note: As both efficiency and line regulation are poor, compensating capacitors must be installed at the receiving-end to transmit this amount of power.

5.4

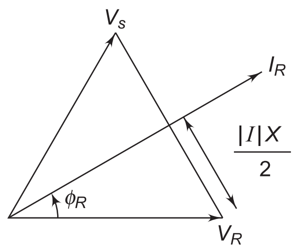


Fig. S-5.4 a

$|V_S| = |V_R| = 230/\sqrt{3} = 132.8$ kV; $\sin\ \phi_R = \dfrac{18\times 1}{2\times 132.8} = 0.068$

$I_R = 998 + j\ 68$ A $\qquad\qquad \cos\ \phi_R = 0.998$

$I_L(\text{load}) = 998 - j\ (998\ \tan\ \cos^{-1}\ 0.85) = 998 - j\ 618.5$

$I_C\ (\text{syn cap}) = j\ (618.5 + 68) = j\ 686.5$

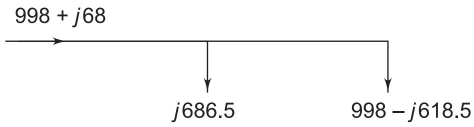


Fig. S-5.4 b

(a) Rating of syn cap = $\sqrt{3} \times 230 \times 0.6865$ = **273.5** MVA

(b) $|I_L|$ = **1,174** A

(c) Load = $\sqrt{3} \times 230 \times 1.174$ = **467.7** MVA

5.5.

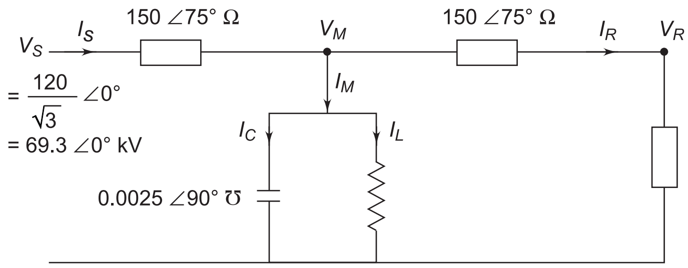


Fig. S-5.5

$$I_s = \frac{40}{\sqrt{3} \times 120} \angle 0^\circ = 0.1925\ \angle 0^\circ \text{ kA}$$

$$V_M = V_S - 150\ \angle 75^\circ\ I_S = 69.3 - 150 \times 0.1925\ \angle 75^\circ = 67.83\ \angle -24.3^\circ \text{ kV}$$

$$I_C = 0.0025 \times 67.83\ \angle 65.7^\circ = 0.17\ \angle 65.7^\circ$$

$$|I_L| = \frac{10}{3 \times 67.83} \qquad \therefore \qquad I_L = 0.049\ \angle 24.3^\circ \text{ kA}$$

$$I_R = I_S - I_C - I_L = 0.193 - 0.17\angle 65.7^\circ - 0.049\angle -24.3^\circ$$
$$= 0.149\angle 7.7^\circ \text{ kA}$$

$$V_R = V_M - 150\angle 75^\circ\ I_R = 67.83\ \angle -24.3^\circ - 28.8\ \angle 9.2^\circ$$
$$= 77.32\angle -4.28^\circ \text{ kV}$$

$|V_R|$ (line) = $\sqrt{3} \times 77.32$ = **133.92** kV

$pf = \cos\ (40.28 + 7.73) = 0.669$ leading

Load = $\sqrt{3} \times 133.92 \times 0.149 \times 0.669$ = **23.12** MW

5.6 Given

$|V_s|$ (line) = 220 kV, $A = 0.93 + j\ 0.016 = 0.93\ \angle 1^\circ$

$B = 20 + j140 = 141.4\ \angle 81.9^o$; $P_R = 60 \times 0.8 = 48$ MW

$Q_R = 60 \times 0.6 = 36$ MVAR;

After substituting these values in Eqs (5.61) and (5.62), we get

$$48 = \frac{220\,|V_R|}{141.4}\cos\,(81.9^o - \delta) - \frac{0.93}{141.4}|V_R|^2 \cos 80.9^o \qquad \text{(i)}$$

$$36 = \frac{220\,|V_R|}{141.4}\ \sin\,(81.9^o - \delta) - \frac{0.93}{141.4}|V_R|^2 \sin 80.9^o \qquad \text{(ii)}$$

or $|V_R| \cos (81.9^o - \delta) = 30.85 + 6.69 \times 10^{-4}\ |V_R|^2$ (iii)

$|V_R| \sin (81.9^o - \delta) = 23.14 + 4.17 \times 10^{-3}\ |V_R|^2$ (iv)

Squaring and adding (iii) and (iv)

$|V_R|^2 = 1487 + 0.2343\ |V_R|^2 + 1784 \times 10^{-8}\ |V_R|^4$

$0.1784 \times 10^{-4}\ |V_R|^4 - 0.7657\ |V_R|^2 + 1487 = 0$

Solving $|V_R|^2 = 4.088 \times 10^4$ (Taking the higher value)

$\therefore \quad |V_R| =$ **202.2** kV

5.7 From Problem 5.3: $Y = 0.00056\ \angle 90^o$, $Z = 87.1\angle 68.4^o$

$V_R = 76.21\ \angle 0^o$ kV; $I_R = 0.2734\ \angle -36.9^o$ kA

$$Z_c = \sqrt{Z/Y} = \sqrt{\frac{87.1}{0.00056}}\ \angle -21.6^o = 394.4\ \angle -10.8^o$$

$$\gamma = \frac{1}{l}\sqrt{YZ} = \frac{1}{200}\sqrt{87.1 \times 0.00056 \angle 158.4^o}$$

$$= 1.104 \times 10^{-3}\ \angle 79.2^o$$

$\therefore \quad \alpha = 0.206 \times 10^{-3}, \qquad \beta = 1.084 \times 10^{-3}$

$$(V_R/Z_c + I_R)/2 = \left(\frac{76.21}{394.4}\angle 10.8^o + 0.2734\angle -36.9^o\right)\Big/2$$

$$= 0.222\ \angle -21.7^o$$

$$(V_R/Z_c - I_R)/2 = 0.083\ \angle 109^o$$

At the receiving-end ($x = 0$)

Incident wave, $i_{x1} = \sqrt{2}\left|\frac{V_R/Z_c + I_R}{2}\right| \cos\,(\omega t + \phi_1)$

$= 0.314 \cos (\omega t - 21.7^o)$

Reflected wave, $i_{x2} = \sqrt{2}\left|\frac{V_R/Z_c - I_R}{2}\right| \cos\,(\omega t + \phi_2)$

$= 0.117 \cos(\omega t + 109^o)$

At 200 km from the receiving-end ($x = 200$)

$$i_{x1} = \sqrt{2}\left|\frac{V_R/Z_c + I_R}{2}\right| e^{\alpha x} \cos(\omega t + \beta x + \phi_1)$$

$$i_{x2} = \sqrt{2}\left|\frac{V_R/Z_c - I_R}{2}\right| e^{-\alpha x} \cos(\omega t - \beta x + \phi_2)$$

$$e^{\alpha x} = e^{0.0412} = 1.042;\ e^{-\alpha x} = e^{-0.0412} = 0.9596$$

$$\beta x = 1.084 \times 10^{-3} \times 200 = 0.2168 \text{ rad} = 12.4^\circ$$

$\therefore$ $i_{x1} = \mathbf{0.327 \cos(\omega t - 9.3^\circ)}$

$i_{x2} = \mathbf{0.112 \cos(\omega t + 96.6^\circ)}$

5.8 $A = \cosh \gamma l = \cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l = 0.93 + j\,0.016$

$\therefore$ $\cosh \alpha l \cos \beta l = 0.93;\ \sinh \alpha l \sin \beta l = 0.016$

or $1 = \dfrac{(0.93)^2}{\cosh^2 \alpha l} + \dfrac{(0.016)^2}{\sinh^2 \alpha l}$

[Exact solution can be obtained numerically]

Let us approximate $\cosh \alpha l = 1 + \alpha^2 l^2/2;\ \sinh \alpha l = \alpha l$

$$\therefore \quad 1 = \frac{(0.93)^2}{\left(1 + \frac{\alpha^2 l^2}{2}\right)^2} + \frac{(0.016)^2}{\alpha^2 l^2}$$

Since αl will be very small for $l = 200$ km; $\left(1 + \dfrac{\alpha^2 l^2}{2}\right)^2 \approx 1.$

$$\therefore \quad \alpha^2 l^2 = \frac{(0.016)^2}{1 - (0.93)^2} \quad \text{or} \quad \alpha l = 0.0435$$

$$\therefore \quad \alpha = 0.0435/200 = 0.218 \times 10^{-3} \text{ rad}$$

(It is a fair approximation)

Now $\cos \beta l = \dfrac{0.93}{\cosh \alpha l}$

$$\cosh \alpha l = (e^{\alpha l} + e^{-\alpha l})/2 = \frac{1.0445 + 0.9574}{2} = 1$$

$$\cos \beta l = 0.93 \ \therefore\ \beta = \cos^{-1} 0.93/200 = 1.882 \times 10^{-3} \text{ rad}$$

$$B = Z_c \sinh \gamma l = 20 + j\,140 = 141.4 \angle 81.9^\circ$$

$$\sinh \gamma l \approx \gamma l = (\alpha + j\beta) l = (0.218 + j\,1.882) \times 0.2 = 0.379 \angle 83.4^\circ$$

$$Z_c = \frac{B}{\sinh rl} = \frac{141.4}{0.379}\ \frac{\angle 81.9^\circ}{\angle 83.4^\circ} = \mathbf{373.1 \angle{-1.5^\circ}}$$

Wave length $\lambda = 2\pi/\beta = 2\pi/1.882 \times 10^{-3} =$ **3,338** km

Velocity of propagation, $v = f\lambda = 50 \times 3{,}338 =$ **166,900** km/sec

Now $A = 0.93 \angle 1^\circ$, $B = 141.4 \angle 81.9^\circ$

$$C = \frac{AD-1}{B} = \frac{0.865\angle 2^\circ - 1}{141.4\angle 81.9^\circ} = 0.001 \angle 85.7^\circ$$

$$V_R = 220/\sqrt{3} \angle 0^\circ = 127 \angle 0^\circ \text{ kV}$$

$$I_R = \frac{50}{\sqrt{3}\times 220\times 0.8} \angle -36.9^\circ = 0.164 \angle -36.9^\circ \text{ kA}$$

$$V_S = 0.93 \angle 1^\circ \times 127 \angle 0^\circ + 141.4 \angle 81.9^\circ \times 0.164 \angle -36.9^\circ$$
$$= \mathbf{135.8 \angle 7.8^\circ} \text{ kV}$$

$$I_S = 0.001 \angle 85.7^\circ \times 127 \angle 0^\circ + 0.93 \angle 1^\circ \times 0.164 \angle -36.9^\circ$$
$$= \mathbf{0.138 \angle 15.6^\circ} \text{ kA}$$

Sending-end power factor = cos (15.6° – 7.8°) = **0.99 leading**

Sending-end power = 3 × 135.8 × 0.138 × 0.99 = **55.66** MW

Transmission efficiency = 50 × 100/55.66 = **89.8%**

5.9 $$Z' = Z\left(\frac{\sin h\,\gamma l}{\gamma l}\right); \; Y'/2 = \frac{Y}{2}\left(\frac{\tan \gamma l/2}{\gamma l/2}\right) = \frac{1}{Z_c}\left(\frac{\cos h\,\gamma l - 1}{\sin h\,\gamma l}\right)$$

$$Z_c = \sqrt{Z/Y} = \sqrt{131.2\angle 72.3^\circ / 10^{-3}\angle 90^\circ} = 362.2 \angle -8.85^\circ \; \Omega$$

As already computed in Example 5.7 (see Text)

$$\gamma l = 0.362 \angle 81.20^\circ; \; \cos h\; \gamma l = 0.938 + j\,0.02 = 0.938 \angle 1.2^\circ$$

$$\sin h\; \gamma l = 0.052 + j\,0.35 = 0.354 \angle 81.5^\circ$$

$$Z' = 131.2 \angle 72.3^\circ \times 0.354 \angle 81.5^\circ / 0.362 \angle 81.2^\circ = \mathbf{128.3\angle 72.6^\circ}$$

$$\frac{Y'}{2} = \frac{1}{362.2\angle -8.85^\circ} \times \frac{0.938 + j0.02 - 1}{0.354\angle 81.5^\circ} = \mathbf{0.00051 \angle 89.5^\circ}$$

5.10

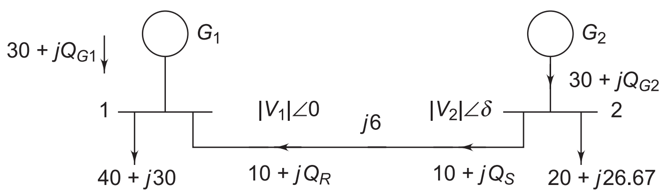


Fig. S-5.10

$$P_{D1} + j\,Q_{D1} = 40 + j\,40 \tan \cos^{-1} 0.8 = 40 + j\,30;$$

$$|V_1| = |V_2| = 22 \text{ kV}$$

$$P_{D2} + j\,Q_{D2} = 20 + j\,20 \tan \cos^{-1} 0.6 = 20 + j\,26.67$$

$$P_S = P_R = \frac{22\times 22}{6} \sin\,\delta = 10 \quad \therefore \sin\,\delta = 60/484$$

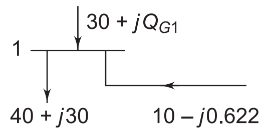
$\therefore \qquad \delta$ = **7.12º**

$$Q_S = -Q_R = |V_2|^2/X - \frac{|V_1||V_2|}{X}\cos\ \delta$$

$$= \frac{22\times 22}{6} - \frac{22\times 22}{6}\ \cos 7.12^\circ = 0.622 \text{ MVAR}$$

At bus 1

Q_{G1} = 30 + 0.622 = 30.622

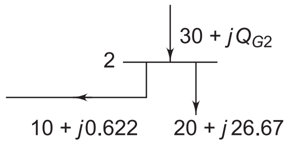


$$pf_1 = \cos\ \tan^{-1}\frac{30.622}{30}$$

= **0.7 lagging**

At bus 2

Q_{G2} = 26.67 + 0.622 = 27.292



$$pf_2 = \cos\ \tan^{-1}\frac{27.292}{30}$$

= **0.74 lagging**

5.11 $R = 400 \times 0.035 = 14\ \Omega;\ X = 314 \times 10^{-3} \times 400 = 125.6\ \Omega$

$Z = R + jX = 14 + j\ 125.6 = 126.4\ \angle 83.6^\circ$

$Y = 314 \times 0.01 \times 10^{-6} \times 400\ \angle 90^\circ = 1.256 \times 10^{-3}\ \angle 90^\circ$

Using nominal-π

$$A = 1 + \frac{1}{2}YZ = 1 + \frac{1}{2} \times 1.256 \times 10^{-3}\ \angle 90^\circ \times 126.4\ \angle 83.6^\circ = 0.921\angle 0.6^\circ$$

$B = Z = 126.4\angle 83.6^\circ$

From Eq. (5.61) we can write

$$P_R = 0 = \frac{(275)^2}{126.4}\cos(83.6^\circ - \delta) - \frac{0.921}{126.4} \times (275)^2\ \cos(83.6^\circ - 0.6^\circ)$$

$\therefore\quad \cos(83.6^\circ - \delta) = 0.921\ \cos 83^\circ = 0.112 \quad \therefore \quad \delta = 0.05^\circ$

From Eq. (5.62)

$$\therefore \quad Q_R = \frac{(275)^2}{126.4}\sin 83.55^\circ - \frac{0.921\times(275)^2}{126.4}\ \sin 83^\circ$$

= **47.56 MVAR** lagging

5.12 $P_D + jQ_D = 2.0 + j\ 2\ \tan\ \cos^{-1} 0.85 = 2.0 + j\ 1.24$

$$\frac{-jQ_C}{P_R + jQ_R}$$

$= -j\ 2.1$

$= 2.0 - j\ 0.86$

$= 2.18$ MVA, 23.3° leading

$pf = 0.918$

$$Z = 3 + j10 = 10.44 \angle 73.3°$$

$$I_R = (2 \cdot 18/\sqrt{3} \times 11) \quad \angle 23.3° = 0.1144 \angle 23.3° \text{ kA}$$

$$V_S = V_R + Z\, I_R$$

$$= 11/\sqrt{3} + 10.44 \angle 73.3° \times 0.1144 \angle 23.3°$$

$$= 6.33 \angle 10.8°$$

$$|V_S| \text{ (line)} = \sqrt{3} \times 6.33 = \mathbf{10.97\ kV}$$

$$I_S = I_R = 0.1144 \angle 23.3° \text{ kA}$$

Sending-end pf = cos 12.50° = **0.98 leading**

Sending-end power = $\sqrt{3} \times 10.97 \times 0.1144 \times 0.98 = 2.13$ MW

$$\eta = \frac{2}{2 \cdot 130} \times 100 = \mathbf{93.9\%}$$

Voltage regulation = (10.97 – 11) × 100/11 = – **0.27%**

5.13 $P_D + j\, Q_D = 30 + j\, 30 \tan \cos^{-1} 0.85 = 30 + j\, 18.59$

$$I_R = \frac{30}{\sqrt{3} \times 33 \times 0.85} \angle -31.8°$$

$$= 0.6175 \angle -31.8° \text{ kA}$$

$$Z = 5 + j\, 20 = 20.62 \angle 76°$$

$$V_S = 33/\sqrt{3} + 20.62 \angle 76° \times 0.6175 \angle -31.8°$$

$$= 29.54 \angle 17.5°$$

$$|V_S| \text{ (line)} = \sqrt{3} \times 29.54 = \mathbf{51.16} \text{ kV}$$

From Eq. (5.66) [$|V_S|$ = 33 kV]

$$P_D = P_R = 30$$

$$= \frac{(33)^2}{20.62} \cos (76° - \delta) - \frac{(33)^2}{20.62} \cos 76°$$

Solving, we get $\delta = 40.1°$

From Eq. (5.67)

$$Q_R = \frac{(33)^2}{20.62} \sin (76° - 40.1°)$$

$$- \frac{(33)^2}{20.62} \sin 76° = -20.28$$

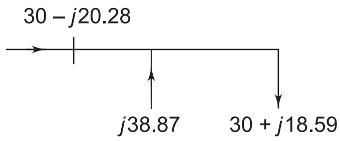


Fig. S-5.13

$$Q_C = -(18.59 + 20.28) = -38.87$$

$$= \mathbf{38.87\ MVAR\ leading}$$

From Eq. (5.66) with $(\theta - \delta) = 0°$

$$P_R(\max) = \frac{(33)^2}{20.62} (1 - \cos 76°) = \mathbf{40\ MW}$$

5.14 $A = 0.938 \angle 1.2°$ $B = 131.2 \angle 72.3°$

Receiving-end circle $\quad OC_R = \dfrac{0.938 \times (220)^2}{131.2} = 346.0$ MVA

$$P_D + j\,Q_D = 50 + j\,50 \tan \cos^{-1} 0.8 = 50 + j\,37.5;\ \theta_R = 36.9°$$

$$\beta - \alpha = 72.3° - 1.2° = 71.1°$$

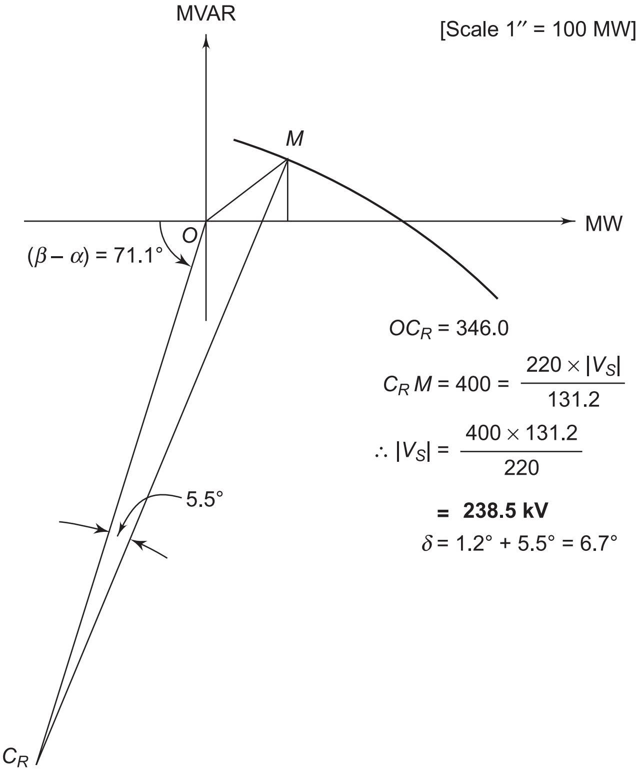


Fig. S-5.14 (a)

Sending-end circle

$$OC_S = \frac{0.938}{131.2} \times (238.5)^2 = 406.6 \text{ MVA}$$

$$\delta + \alpha = 6.7° + 1.2° = 7.9°$$

$$P_S + j\,Q_S = 53 - j\,10$$

$$pf = \cos \tan^{-1} \frac{10}{53}$$

= **0.983 leading**

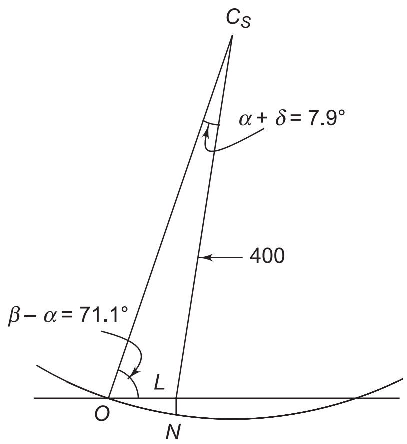


Fig. S-5.14 (b)

5.15 $Z = 5 + j\,25 = 25.5\ \angle 78.7°$

$$P_D + j\,Q_D = 15 + j\,15 \tan \cos^{-1} 0.8 = 15 + j\,11.25$$

$$P_R = P_D = 15 = \frac{(33)^2}{25.5} \cos (78.7° - \delta)\ \frac{(33)^2}{25.5} \cos 78.7°$$

$$\cos (78.7° - \delta) = \frac{25.5}{(33)^2} \times 15 + \cos 78.7°$$

$$\therefore \qquad \delta = 21.9°$$

$$Q_R = \frac{(33)^2}{25.5} \sin (78.7° - 21.9°) - \frac{(33)^2}{25.5} \sin 78.7°$$

$$= \frac{(33)^2}{25.5} [\sin 56.8° - \sin 78.7°] = -6.14$$

$\therefore \qquad Q_C =$ **17.39 MVAR leading**

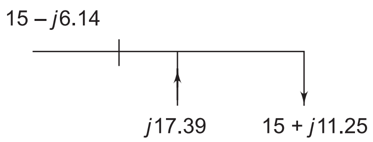


Fig. S-5.15

Now $\qquad |V_R| = 28$ kV

$$P_D + j\,Q_D = P_D (1 + j \tan \cos^{-1} 0.8)$$

$$= P_D (1 + j\,0.75)$$

$$P_R + j\,Q_R = P_D + j\,(0.75\ P_D - 17.39)$$

$$P_R = P_D = \frac{33 \times 28}{25.5} \cos (78.7° - \delta) - \frac{(28)^2}{25.5} \cos 78.7°$$

$$0.75\ P_D - 17.39 = \frac{33 \times 28}{25.5} \sin (78.7° - \delta) = \frac{(28)^2}{25.5} \sin 78.7°$$

$$\text{or} \cos (78.7° - \delta) = \frac{25.5}{33 \times 28} P_D + \frac{28}{33} \cos 78.7° = 0.0276\ P_D + 0.1663$$

$$\sin (78.7° - \delta) = \frac{25.5 \times 0.75}{33 \times 28} P_D - \frac{25.5 \times 17.39}{33 \times 28} + \frac{28}{33} \sin 78.7°$$

$$= 0.0207\ P_D + 0.352$$

Squaring and adding

$$1 = 1.19 \times 10^{-3} P_D^2 + 23.7 \times 10^{-3} P_D + 0.1516$$

or

$$P_D^2 + 19.92\ P_D - 0.713 \times 10^3 = 0$$

$$P_D = \frac{-19.92 \pm \sqrt{(19.92)^2 + 2.852 \times 10^3}}{2}$$

$$= 18.54 \text{ MW (negative solution is rejected)}$$

Extra power transmitted = 18.54 – 15 = **3.54 MW**

Note: It is assumed in this problem that as the receiving-end voltage drops, the compensating equipment draws the same MVAR (leading).

Chapter 6

6.1

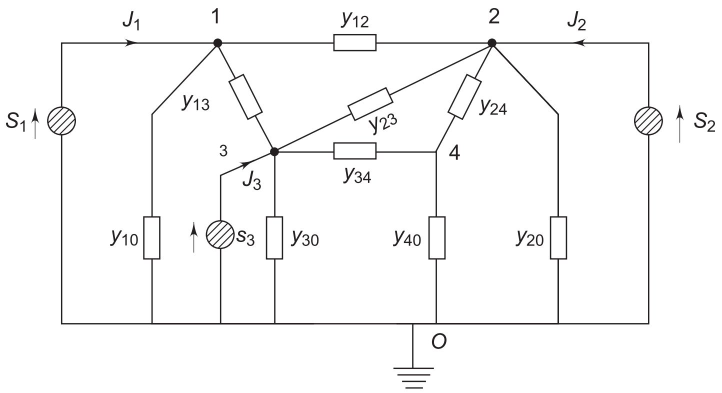


Fig. S-6.1(a)

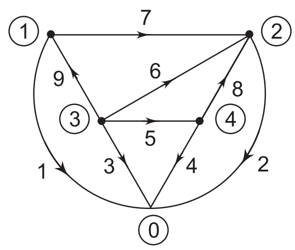


Fig. S-6.1(b) Linear graph of the circuit of Fig. S-6.1 a

For this network tree is shown in Fig. 6.3 (a) and hence A is given by Eq. (6.17).

This matrix is not unique. It depends upon the orientation of the elements.

6.2

$$Y_{BUS} = \begin{array}{c} \begin{matrix} \quad 1 \quad & \quad\quad 2 \quad\quad & \quad 3 \quad \end{matrix} \\ \left[\begin{array}{c|c|c} \dfrac{1}{0.04 + j0.06} & \dfrac{-1}{0.04 + j0.06} & 0 \\ \hline \dfrac{-1}{0.04 + j0.06} & \dfrac{1}{0.04 + j0.06} + \dfrac{1}{0.02 + j0.03} & \dfrac{-1}{0.02 + j0.03} \\ \hline 0 & \dfrac{-1}{0.02 + j0.03} & \dfrac{1}{0.02 + j0.03} \end{array}\right] \end{array}$$

$$\therefore \qquad Y_{BUS} = 27.735 \angle -56.3° \begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 1.5 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

From Eq. (6.45)

$$V_2^1 = \frac{A_2}{(V_2^0)^*} - B_{21}\, V_1 - B_{23}\, V_3^0$$

Here $A_2 = \dfrac{P_2 - jQ_2}{Y_{22}} = \dfrac{-5.96 - j1.46}{41.602 \angle -56.3°}$

$B_{21} = \dfrac{Y_{21}}{Y_{22}} = \dfrac{-13.867 \angle -56.3°}{41.602 \angle -56.3°}$; $B_{23} = \dfrac{Y_{23}}{Y_{22}} = \dfrac{-27.735}{41.602}$

$\therefore \quad V_2^1 = \dfrac{-5.96 - j1.46}{41.602 \angle -56.3°} + \dfrac{13.867}{41.602} + \dfrac{27.735}{41.602} \times 1.02$

$= 0.963 - j\ 0.138 = \mathbf{0.972 \angle -8.15°}$

6.3

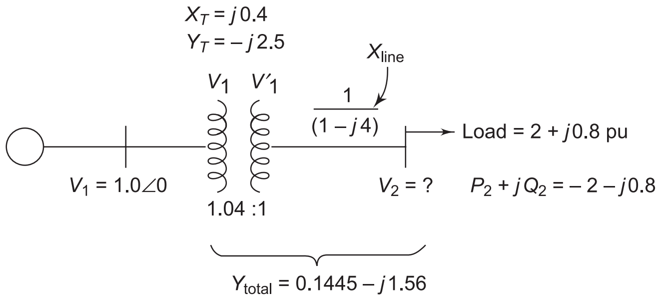


Fig. S-6.3

$$Y_{BUS} = \begin{bmatrix} 0.1445 - j1.56 & -0.1445 + j1.56 \\ -0.1445 + j1.56 & 0.1445 - j1.56 \end{bmatrix}; \ \alpha = 1/1.04$$

$$\text{Modified } Y_{BUS} = \left[\begin{array}{c:c} \dfrac{1}{(1.04)^2}(0.1445 - j1.56) & \dfrac{1}{1.04}(-0.1445 + j1.56) \\ \hdashline \dfrac{1}{1.04}(-0.1445 + j1.56) & 0.1445 - j1.56 \end{array}\right]$$

$$V_2^1 = \frac{-2 + j0.8}{0.1445 - j1.56} - \frac{1}{(1.04)} \frac{-0.1445 + j1.56}{0.1445 - j1.56}$$

$= 0.335 - j\ 1.222 = 1.26 \angle -74.66°$

6.4 Z (series) $= 0.1 + j\ 0.7\ \Omega$/km

(a) Y (shunt) $= j\ 0.35 \times 10^{-5}$ ℧/km

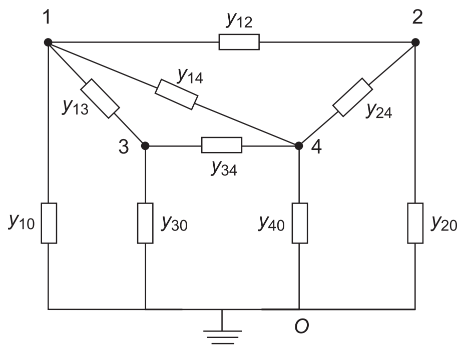


Fig. S-6.4 (a)

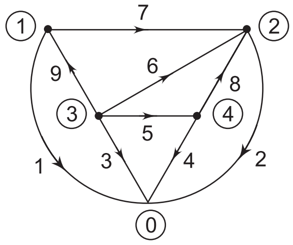


Fig. S-6.4 (b) Linear Graph

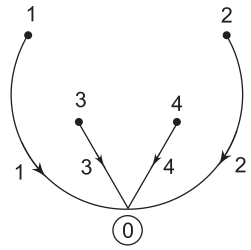


Fig. S-6.4 (c) TREE

$A =$

e \ bus	①	②	③	④
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	0	0	1	−1
6	−1	0	0	1
7	1	−1	0	0
8	0	−1	0	1
9	−1	0	1	0

(b) Base MVA = 100, Base kV = 220

$$\frac{Y_{pu}}{2} \text{ (shunt)} = j\ \frac{0.35}{2} \times 10^{-5} \times \frac{(220)^2}{100} = j\ 84.7 \times 10^{-5}/\text{km}$$

$$Z_{pu} \text{ (series)} = (0.1 + j\ 0.7) \times \frac{100}{(220)^2} = (2.066 + j\ 14.463) \times 10^{-4}/\text{km}$$

$$Y_{pu} \text{ (series)} = \frac{1}{Z_{pu} \text{ (series)}} = (96.8 - j\ 677.6)/\text{km}$$

The permitive admittance matrix (diagonal matrix) for the system will be

$$Y = \begin{bmatrix} y_{10} = j\ 84.7 \times 10^{-5}\ (100 + 110 + 150) = j\ 0.3049 \\ y_{20} = j\ 84.7 \times 10^{-5}\ (100 + 100) = j\ 0.1694 \\ y_{30} = j\ 84.7 \times 10^{-5}\ (110 + 120) = j\ 0.1948 \\ y_{40} = j\ 84.7 \times 10^{-5}\ (100 + 120 + 150) = j\ 0.3134 \\ y_{34} = (96.8 - j\ 677.6)/120 = 0.807 - j\ 5.65 \\ y_{14} = (96.8 - j\ 677.67)/150 = 0.645 - j\ 4.517 \\ y_{12} = (96.8 - j\ 677.6)/100 = 0.968 - j\ 6.776 \\ y_{24} = (96.8 - j\ 677.6)/100 = 0.968 - j\ 6.776 \\ y_{13} = 96.8 - j\ 677.6/110 = 0.880 - j\ 6.160 \end{bmatrix}$$

$Y_{BUS} = A^T YA$

$$= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 2.493 - j17.148 & -0.968 + j6.776 & -0.880 + j6.16 & -0.645 + j4.517 \\ -0.968 + j6.776 & 1.936 - j13.383 & 0 & -0.968 + j6.776 \\ -0.880 + j6.160 & 0 & 1.687 - j11.615 & -0.807 + j5.65 \\ -0.645 + j4.517 & -0.968 + j6.776 & -0.807 + j5.650 & 2.42 - j16.63 \end{bmatrix} \end{array}$$

6.5 $P_{G1} = 0.6$; unknowns are δ_2, δ_3, Q_{G1}, Q_{G2} and Q_{G3}.

$$Y_{BUS} = \begin{bmatrix} -j10 & j5 & j5 \\ j5 & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

From Eq. (6.37) after substituting the relevant data ($\delta_1 = 0$) we get

$$1.4 = 10\ \delta_2 - 5\delta_3;\ -1 = -5\delta_2 + 10\delta_3$$

which give

$$\delta_2 = 0.12 \text{ rad} = 6.87°,\ \ \delta_3 = -0.04 \text{ rad} = -2.29°$$

Substituting the various values and values of δ_2 and δ_3 in Eq. (6.38) and solving we get

$$Q_1 = 0.040 \text{ pu},\ Q_2 = 0.100 \text{ pu};\ Q_3 = 0.068 \text{ pu}$$

∴ Reactive power generations at the three buses are

$$Q_{G1} = Q_1 + 0.6 = 0.640 \text{ pu}$$
$$Q_{G2} = Q_2 = 0.100 \text{ pu};\ Q_{G3} = Q_3 + 1 = 1.068 \text{ pu}$$

Reactive losses on the three lines are

$$Q_L = \sum_{i=1}^{3} Q_{Gi} - \sum_{i=1}^{3} Q_{Di} = 1.808 - 1.6 = 0.208 \text{ pu}$$

Using Eq. (5.71) we can find real power flows as:

$$P_{12} = \frac{1}{0.2} \sin(-6.87°) = \mathbf{-0.598\ pu}$$

$$P_{13} = \frac{1}{0.2} \sin 2.29° = \mathbf{0.200\ pu} \text{ (Notice } P_{ik} = -P_{Ri})$$

$$P_{23} = \frac{1}{0.2} \sin 9.16° = \mathbf{0.796\ pu}$$

For reactive power flows Eq. (5.69) is used.

$$Q_{12} = Q_{21} = \frac{1 - \cos(-6.87°)}{0.2} = \mathbf{0.036\ pu}$$

$$Q_{13} = Q_{31} = \frac{1 - \cos 2.29}{0.2} = \mathbf{0.004} \text{ pu}$$

$$Q_{23} = Q_{32} = \frac{1 - \cos 9.16°}{0.2} = \mathbf{0.064} \text{ pu}$$

Various line flaws are indicated in Fig. S-6.5.

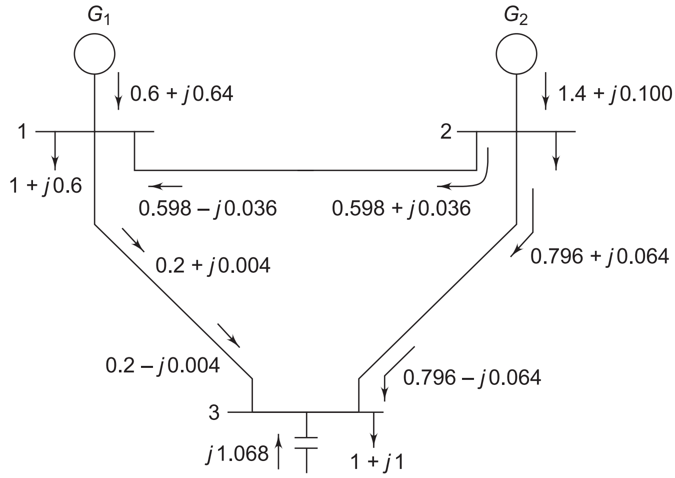


Fig. S-6.5 Load flow solution for the sample system

6.6 (a) $|V_1| = 1$ pu, $|V_2| = 1.04$ pu $|V_3| = 0.96$ pu; $P_{G1} = 0.6$ pu
$\delta_1 = 0$ Substituting the data in Eq. (6.37) we get

$$1.4 = 1.04 \times 5\ \delta_2 + 1.04 \times 0.96 \times 5\ (\delta_2 - \delta_3)$$

$$-1 = 0.96 \times 5\ \delta_3 + 1.04 \times 0.96 \times 5\ (\delta_3 - \delta_2)$$

Simplifying, and solving we get

$$\delta_2 = 0.1164 \text{ rad} = 6.66°;\ \delta_3 = -0.0427 \text{ rad} = -2.45°$$

Substituting the values of various quantities in Eq. (6.38) and solving

$$Q_1 = 0.0395 \text{ pu},\ Q_2 = 0.722 \text{ pu},\ Q_3 = -0.508 \text{ pu}$$

$$\therefore\ Q_{G1} = 0.64,\ Q_{G2} = 0.722,\ Q_{G3} = 0.492 \text{ pu}$$

$$Q_L = \Sigma\, Q_{Gi} - \Sigma\, Q_{Di} = 1.854 - 1.6 = 0.254 \text{ pu}$$

Real line flows

$$P_{ik} = -P_{ki} = \frac{|V_i||V_k|}{X_{ik}} \sin\ \delta_{ik}$$

$$P_{12} = \frac{1}{0.2} \sin(-\delta_2) = -\mathbf{0.58} \text{ pu} = -5 \sin 6.66°$$

$$P_{13} = \frac{1}{0.2} \sin(-\delta_3) = 5 \sin 2.45° = \mathbf{0.214} \text{ pu}$$

$$P_{23} = \frac{1}{0.2} \sin(\delta_2 - \delta_3) = 5 \sin 9.11° = \mathbf{0.792} \text{ pu}$$

$$Q_{ik} = \frac{|V_i|^2}{X_{ik}} - \frac{|V_i||V_k|}{X_{ik}} \cos \delta_{ik}$$

∴ Reactive power flows:

$$Q_{12} = \frac{1}{0.2} - \frac{1 \times 1.04}{0.2} \cos(-6.66°) = -\mathbf{0.165} \text{ pu}$$

$Q_{21} = \mathbf{0.243}$ pu; $Q_{13} = \mathbf{0.204}$ pu

$Q_{31} = -\mathbf{0.188}$ pu; $Q_{23} = \mathbf{0.479}$ pu; $Q_{32} = -\mathbf{0.321}$ pu

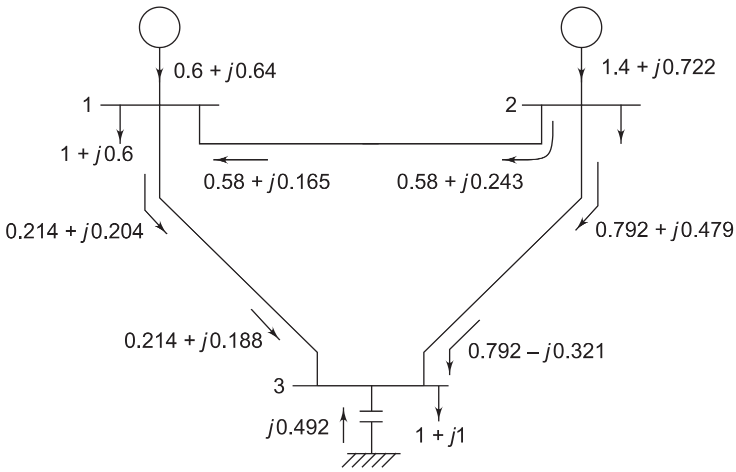


Fig. S-6.6 (a) Load flow solution for the sample system of Problem 6.6 a

It immediately follows from the load flows of Problems 6.5 and 6.6 (a) that there is no significant change in real power flows but the reactive power flows have changed significantly.

(b) $|V_1| = |V_2| = |V_3| = 1.0$ pu; $P_{G1} = P_{G2} = 1.0$ pu, $P_{G3} = 0$

$\delta_1 = 0$, From Eq. (6.37), substituting

$P_2 = 1.0$ and $P_3 = -1$, we get

$1 = 10\ \delta_2 - 5\ \delta_3$ and $-1 = -5\ \delta_2 + 10\ \delta_3$

Solving $\delta_2 = -0.0667$ rad $= 3.82°$

$\delta_3 = -0.0667$ rad $= -3.82°$

Substituting the values of δ_2 and δ_3 in Eq. (6.38) we get

$Q_1 = -0.022$ pu; $Q_2 = 0.055$ pu

$Q_{G1} = Q_1 + 0.6 = 0.622$ pu, $Q_{G2} = Q_2 = 0.055$ pu

$Q_{G3} = Q_3 + 1 = 1.055$ pu, $Q_L = 1.732 - 1.6 = 0.132$ pu

Real line flows

P_{12} = – 0.333 pu, P_{13} = **0.333** pu

P_{23} = **0.664** pu

Reactive line flows

$$Q_{12} = Q_{21} = \frac{1 - \cos(-3.82°)}{0.2} = \mathbf{0.011} \text{ pu}$$

$$Q_{13} = Q_{13} = \frac{1 - \cos 3.82}{0.2} = \mathbf{0.011} \text{ pu}$$

$$Q_{23} = Q_{32} = \mathbf{0.044} \text{ pu}$$

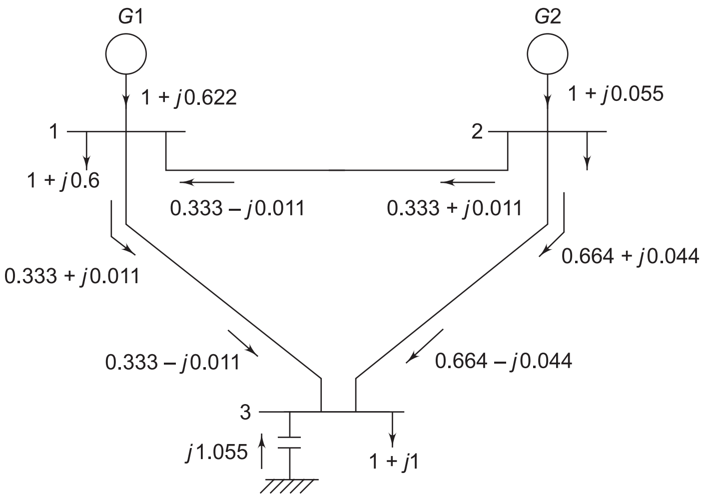


Fig. 6.6 (b) Load flow solution for the sample system

It is noticed from the load flows of Problems 6.5 and 6.6 (b) that while there are significant changes in real power flows, the changes in reactive power flows are much smaller.

6.7 (a) (i) $V_1/V_1' = 0.99$ or $\alpha = 1/0.99$

$$Y_{BUS}\text{, modified} = \begin{bmatrix} -j5[1 + 1/(0.99)^2] = -j10.1015 & j5/0.99 = j5.0505 & j5 \\ j5.0505 & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

(ii) $\alpha = e^{-j3°}$

$$Y_{BUS}\text{, modified} = \begin{bmatrix} -j10 & j5\,e^{j3°} = 5\angle 93° & j5 \\ j5\,e^{-j3°} = 5\angle 87° & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

(b) $P_2 = 1.4 = 5.0505\ \delta_2 + 5\ (\delta_2 - \delta_3)$

$$P_3 = -1 = 5\ \delta_3 + 5\ (\delta_3 - \delta_2)$$

Solving we get

$$\delta_2 = 0.119 \text{ rad} = 6.82°;$$
$$\delta_3 = -0.0405 \text{ rad} = -2.32°$$

$$Q_1 = -5.0505 \cos(-6.82°) - 5 \cos 2.32° + 10.10152 = 0.091 \text{ pu}$$
$$Q_2 = -5.0505 \cos 6.82° - 5 \cos 9.14° + 10 = 0.049 \text{ pu}$$
$$Q_3 = -5 \cos(-2.32°) - 5 \cos 9.14° + 10 = 0.068 \text{ pu}$$
$$Q_{G1} = 0.691 \text{ pu},\ Q_{G2} = 0.049 \text{ pu},\ Q_{G3} = 1.068 \text{ pu}$$
$$Q_L = 1.808 - 1.6 = 0.208 \text{ pu}$$
$$P_{12} = \mathbf{0.600} \text{ pu},\ P_{13} = \mathbf{0.202} \text{ pu},\ P_{23} = \mathbf{0.794} \text{ pu}$$
$$Q_{12} = \frac{(1/0.99)^2}{0.2} - \frac{1/0.99}{0.2} \cos -6.82° = \mathbf{0.087} \text{ pu}$$
$$Q_{21} = \frac{1}{0.2} - \frac{1/0.99}{0.2} \cos 6.82° = -\mathbf{0.014} \text{ pu}$$
$$Q_{13} = Q_{31} = 0.004 \text{ pu};\ Q_{23} = Q_{32} = \mathbf{0.064} \text{ pu}$$

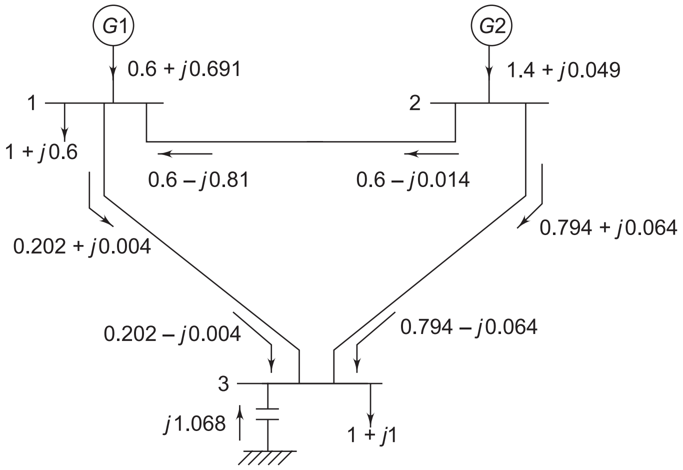
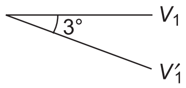


Fig. S-6.7 (a) Load flow solution for $\alpha = 1/0.99$

Remark: Only the reactive flow on the line with regulating transformer is changed.



Case (ii) $\alpha = e^{-j3°}$

$$\delta_1' = -3° \text{ or } -0.0523 \text{ rad.}$$
$$P_2 = |Y_{21}|\ (\delta_2 - \delta_1 + 3°)\ (0.0523 \text{ rad})) + |Y_{23}|\ (\delta_2 - \delta_3)$$
$$P_3 = |Y_{31}|\ (\delta_3 - \delta_1) + |Y_{32}|\ (\delta_3 - \delta_2)$$

$$1.4 = 5\,(\delta_2 + 0.0523) + 5\,(\delta_2 - \delta_3)$$
$$-1 = 5\,\delta_3 + 5\,(\delta_3 - \delta_2)$$

Solving we get

$$\delta_2 = 0.0852 \text{ rad} = 4.88°;\ \ \delta_3 = -0.057 \text{ rad} = -3.29°$$
$$Q_1 = -|Y_{12}|\cos(\delta_1 - \delta_2 - 3°) - |Y_{13}|\cos(\delta_1 - \delta_3) + |Y_{11}|$$
$$= -5\cos(-7.88°) - 5\cos 3.29° + 10 = 0.055 \text{ pu}$$
$$Q_2 = -|Y_{21}|\cos(\delta_2 - \delta_1 + 3°) - |Y_{23}|\cos(\delta_2 - \delta_3) + |Y_{22}|$$
$$= 0.098 \text{ pu}$$
$$Q_3 = 0.059 \text{ pu}$$
$$Q_{G1} = 0.655 \text{ pu},\ \ Q_{G2} = 0.098 \text{ pu},$$
$$Q_{G3} = 1.059 \text{ Pu},\ \ Q_L = 0.212 \text{ pu}$$

Real line flows

$$P_{12} = 5\sin(\delta_1' - \delta_2) = -5\sin 7.88° = -\mathbf{0.685} \text{ pu}$$
$$P_{13} = 5\sin(\delta_1 - \delta_3) = 5\sin 3.29° = \mathbf{0.287} \text{ pu}$$
$$P_{23} = 5\sin(\delta_2 - \delta_3) = 5\sin 8.17° = \mathbf{0.711} \text{ pu}$$

Reactive line flows

$$Q_{12} = 5 - 5\cos(\delta_1' - \delta_2) = 5\,(1 - \cos 7.88°) = \mathbf{0.047} \text{ pu}$$
$$Q_{13} = 5 - 5\cos(\delta_1 - \delta_3) = \mathbf{0.008} \text{ pu};\ \ Q_{23} = \mathbf{0.051} \text{ pu}$$

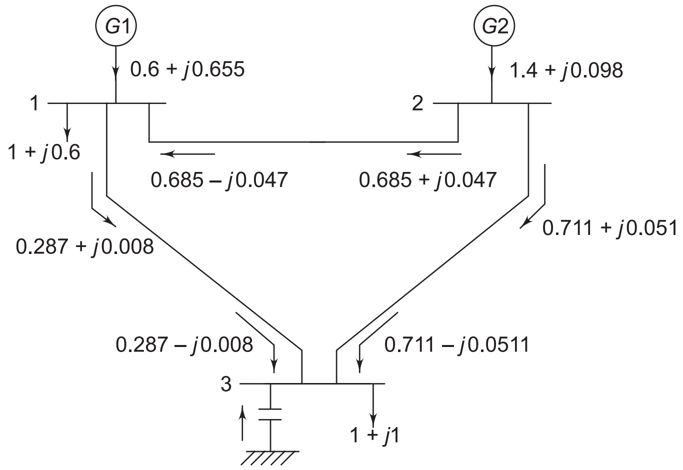


Fig. S-6.7 (b) Load flow solution for the sample system $\alpha = je^{-3°}$

Remark: With introduction of phase shifting transformer in line 1–2, the real load flow changes much more than the changes in reactive load flows.

6.8

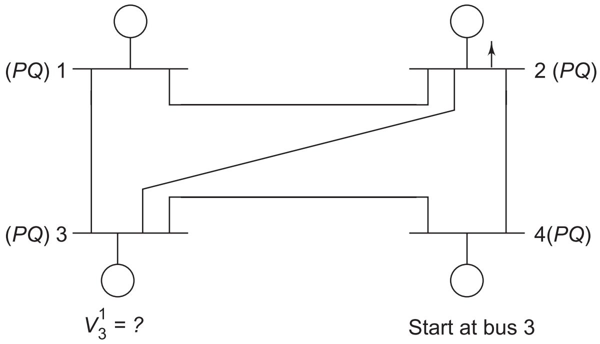


Fig. S-6.8

Refer to Ex. 6.4

$$V_3^1 = \frac{1}{Y_{33}}\left\{\frac{P_3 - j\,Q_3}{(V_3^0)^*} - Y_{31}\,V_1 - Y_{32}\,V_2^0 - Y_{34}\,V_4^0\right\}$$

$$= \frac{1}{Y_{33}}\left\{\frac{-1 - j\,0.5}{1\angle 0^\circ} - 1.04\,(-1 + j\,3) - (-0.666 + j\,2) - (-2 + j\,6)\right\}$$

$$= \frac{1}{3.666 - j11}\left\{2.706 - j\,11.62\right\}$$

$= 1.025 - j\ 0.095$ pu

$=$ **1.029** $\angle$**– 5.3°** pu

Chapter 7

7.1 Data of Ex. 7.2; $P_{G1} = P_{G2} = 110$ MW

From Table 7.1 of the text, for a load of 220 MW optimum schedule is $P_{G1} = 100$ MW, $P_{G2} = 120$ MW

Increase in cost for unit 1 is

$$\int_{100}^{110} (0.2\ P_{G1} + 40)\ dP_{G1} = (0.1\ P_{G1}^2 + 40\ P_{G1})\Big|_{100}^{110} = 610 \text{ Rs/hr}$$

For unit 2

$$\int_{120}^{110} (0.25\ P_{G2} + 30)\ dP_{G2} = -\ 587.5$$

∴ Extra cost incurred in Rs/hr = 610 – 587.5 = **22.5**

7.2 (a)
$$P_{G1} + P_{G2} = 300 \qquad \text{(i)}$$
$$0.1\ P_{G1} + 20 = 0.12\ P_{G2} + 15 \qquad \text{(ii)}$$

Solving (i) and (ii) we get,

$$P_{G1} = 310/2.2 = \mathbf{140.9} \text{ MW}$$

$$P_{G2} = 300 - 140.9 = \mathbf{159.1} \text{ MW}$$

(b) Equal load sharing results in $P_{G1} = P_{G2} = 150$ MW

Increase in cost for unit 1

$$\int_{140.9}^{150} (0.1\ P_{G1} + 20)\ dP_{G1} = 314.36 \text{ Rs/hr}$$

Increase in cost for unit 2

$$\int_{159.1}^{150} (0.12\ P_{G2} + 15)\ dP_{G2} = -\ 305.27 \text{ Rs/hr}$$

$$\text{Net saving} = (314.36 - 305.27) \times 24$$
$$= \textbf{Rs 218.16/day}$$

7.3 (i) Gen. *A* will share more load than Gen. *B*.

(ii) Gen. *A* and Gen. *B* will share load of P_G each.

(iii) Gen. *B* will share more load then Gen. *A*.

7.4 $P_{G1} + P_{G2} + P_{G3} = 400$

$$P_{G1} = -\ 100 + 50\ (IC) - 2\ (IC)^2 \qquad \text{(i)}$$
$$P_{G2} = -\ 150 + 60\ (IC) - 2.5\ (IC)^2 \qquad \text{(ii)}$$
$$P_{G3} = -\ 80 + 40\ (IC) - 1.8\ (IC)^2 \qquad \text{(iii)}$$

Adding (i), (ii) and (iii), we get

$$400 = -\ 330 + 150\ (IC) - 6.3\ (IC)^2$$

or $\quad 6.3\ (IC)^2 - 150\ (IC) + 730 = 0$

∴ $\quad IC = 6.821;\ 16.989$

For $\quad IC = 6.821, \rightarrow P_{G1} = \mathbf{148.0,}$

$P_{G2} = \mathbf{142.9,}\ P_{G3} = \mathbf{109.1}$ MW

For $IC = 16.989 \rightarrow P_{G1} = 172.2,$

$$P_{G2} = 147.8,\ P_{G3} = 80.0 \text{ MW}$$

One of the solutions will be rejected in accounting for the upper and lower limits of machine loading. Here we reject the second solution.

Note: Since the equations are quadratic in (IC), exact solution is possible here.

7.5 Fuel cost = Rs 2/million kilocalories

$$\therefore \quad C = 0.0002\ P_G^3 + 0.06\ P_G^2 + 24.0\ P_G + 300$$

$$\therefore \quad \frac{dC}{dP_G} = 0.0006\ P_G^2 + 0.12\ P_G + 24$$

A plot of $\frac{dC}{dP_G}$ Vs P_G shows a good linear approximation between 0 and 80 MW to be

$$\frac{dC}{dP_G} = 0.175\ P_G + 23$$

7.6 Equation (7.31) for plant 1 becomes

(a) $0.02\ P_{G1} + 2\lambda\ B_{11}\ P_{G1} + 2\lambda\ B_{12}\ P_{G2} = \lambda - 16$

For $\lambda = 26$, $0.02\ P_{G1} + 52 \times 0.001\ P_{G1} = 10\ \therefore\ P_{G1} =$ **138.89** MW

Similarly for plant 2, $0.04\ P_{G2} = 6$ or $P_{G2} =$ **150** MW

Now $P_L = 0.001 \times (138.89)^2 = 19.29$ MW

$\therefore$ $P_D = P_{G1} + P_{G2} - P_L =$ **269.60** MW

(b) $0.02\ P_{G1} + 16 = 0.04\ P_{G2} + 20$

$$P_{G1} + P_{G2} = 0.001\ P_{G1}^2 + 269.61$$

Solving, $P_{G1} =$ **310.8** MW; $P_{G2} =$ **55.4** MW

(c) For part (a)

$$C_T = 0.01\ (138.89)^2 + 16 \times 138.89 + 250 + 0.02\ (150)^2 + 20 \times 150 + 350$$

= Rs **6,465.14/.hr**

For part (b)

$$C_T = 0.01\ (310.8)^2 + 16 \times 310.8 + 250 + 0.02\ (55.4)^2 + 20 \times 55.4 + 350$$

= Rs **7,708.15/hr**

7.7 $I_a = 2 - j\ 0.5$ pu, $I_b = 1.6 - j\ 0.4$ pu, $I_c = 1.8 - j\ 0.45$ pu

$$Z_a = 0.06 + j\ 0.24 \text{ pu},\ Z_b = Z_c = 0.03 + j\ 0.12 \text{ pu}$$

$$\left|\frac{I_C}{I_b + I_c}\right| = \left|\frac{1.8 - j\,0.45}{3.4 - j\,0.85}\right| = 0.5294$$

$$\therefore \quad M_{a1} = -\ 0.5294,\ M_{b1} = 0.4706,\ M_{c1} = 0.5294$$

$$M_{a2} = 0.4706,\ M_{b2} = 0.4706,\ M_{c2} = 0.5294$$

$$V_1 = 1.0 \angle 0° \text{ pu}$$
$$V_2 = 1 + (2 - j\,0.5)\,(0.06 + j\,0.24) = 1.319 \angle 20°$$

The current phase angles at the plants are

$(I_1 = I_b - I_a,\ I_2 = I_a + I_c)$

$$\sigma_1 = \tan^{-1}(0.1/-0.4) = 166°;\quad \sigma_2 = \tan^{-1}\frac{-0.95}{3.8} = -14°$$

$\cos(\sigma_2 - \sigma_1) = -1$

The plant power factors are

$$pf_1 = \cos 166° = -0.97;\quad pf_2 = \cos(20° + 14°) = 0.829$$

From Eq. (7.42)

$$B_{11} = \frac{0.06(0.5294)^2 + 0.03[(0.4706)^2 + (0.5294)^2]}{(-0.97)^2} = \mathbf{0.03387} \text{ pu}$$

$$B_{22} = \frac{0.06 \times (0.4706)^2 + 0.03[(0.4706)^2 + (0.5294)^2]}{(1.319)^2 \times (0.829)^2} = \mathbf{0.0237} \text{ pu}$$

$$B_{12} = \frac{-1\{-0.06 \times 0.5294 \times 0.4706 + 0.03[(0.4706)^2 + (0.5294)^2]\}}{1 \times 1.319 \times (-0.97) \times 0.829}$$

$$= \mathbf{9.6073 \times 10^{-5}} \text{ pu}$$

For a base of 100 MVA

$B_{11} = \mathbf{0.03387 \times 10^{-2}}$ MW^{-1}; $B_{22} = \mathbf{0.0237 \times 10^{-2}}$ MW^{-1}

$B_{12} = \mathbf{9.6073 \times 10^{-7}}$ MW^{-1}

7.8 Economically optimum unit commitment is obtained below by referring to Table 7.3.

Time	*Load MW*	*Unit number*			
		1	*2*	*3*	*4*
0–4	20	1	1	1	1
4–8	14	1	1	1	0
8–12	6	1	1	0	0
12–16	14	1	1	1	0
16–20	4	1	0	0	0
20–24	10	1	1	0	0

Optimal and secure UC Table

In the above table the modification will take place in the last but one row as follows:

16–20	*4*	*1*	*1**	*0*	*0*

* = unit started due to security considerations.

7.9 Load cycle 6 AM – 6 PM ⋯ 220 MW

6 PM – 6 AM ⋯ 40 MW

For 220 MW, referring to Table 7.1 we get $P_{G1} = 100$; $P_{G2} = 120$ MW.

Total fuel cost for this period is = Rs 1,27,440 = 00 (See Ex. 7.3) If both units operate in the light load period also, then Table 7.1 gives P_{G1} = 20 MW; P_{G2} = 20 MW

$$C_T = (0.1 \times 20^2 + 40 \times 20 + 120 + 0.125 \times 20^2 + 30 \times 20 + 100) \times 12$$
$$= \text{Rs } 20{,}520.00$$

Total fuel cost when both units are operating throughout = Rs 1,47,960. If only one of the units is run during the light load period, it is easily verified that it is economical to run unit 2 and to put off unit 1. When the total fuel cost during this period = $(0.125 \times 40^2 + 30 \times 40 \times 100) \times 12$ = Rs 18,000

Total fuel cost = Rs 1,45,440

Total operating cost for this case = 1,45,440 + 400 = Rs 1,45,840

Comparing, we can say it is economical to remove unit 1 from service for the 12 hours of light load period.

7.10 Inequality constraints are considered employing penalty functions. Modified Lagrangian of Eq. (7.77) becomes

$$\mathscr{L} = \sum_m \big[C\,(P_{GT}^m) + W\,(P_{GT}^m) + W(X^m) + W(P_{GH}^m)$$
$$- \lambda_1^m\,(P_{GT}^m + P_{GH}^m - P_L^m - P_D^m) + \lambda_2^m\,(X^m - X^{m-1} - J^m + q^m)$$
$$+ \lambda_3^m\,\{P_{GH}^m - h_0\,(1 + 0.5\;e\,(X^m + X^{m-1}))\,(q^m - e)\big] \quad \text{(i)}$$

where $W(X)$ is a Powell's penalty function of X.

The dual variables are obtained from the equations

$$\frac{\partial \mathscr{L}}{\partial P_{GH}^m} = \frac{dc\,(P_{GT}^m)}{d\,P_{GT}^m} + W'(P_{GT}^m) - \lambda_1^m\left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m}\right) = 0 \quad \text{(ii)}$$

$$\frac{\partial \mathscr{L}}{\partial P_{GT}^m} = W'(P_{GH}^m) + \lambda_3^m - \lambda_1^m\left(1 - \frac{\partial P_L^m}{\partial P_{G11}^m}\right) = 0 \quad \text{(iii)}$$

$$\left(\frac{\partial \mathscr{L}}{\partial \lambda^m}\right)_{\substack{m \neq M \\ \neq 0}} = W'\,(X^m) + \lambda_2^m - \lambda_2^{m+1} - \lambda_3^m\,\{0.5 h_0\;e\;(q^m - \partial)\}$$
$$- \lambda_3^{m+1}\,\{0.5\;h_0\;e\;(q^{m+1} - \rho)\} = 0 \quad \text{(iv)}$$

$$\left(\frac{\partial \mathscr{L}}{\partial q^1}\right) = \lambda^1{}_2 - \lambda_3^1\;h_0\;\{1 + 0.5\;e\;(2X^\circ + J' - 2q' + \rho)\} = 0 \quad \text{(v)}$$

The gradient vector is given by Eq. (7.82)

Chapter 8

8.1

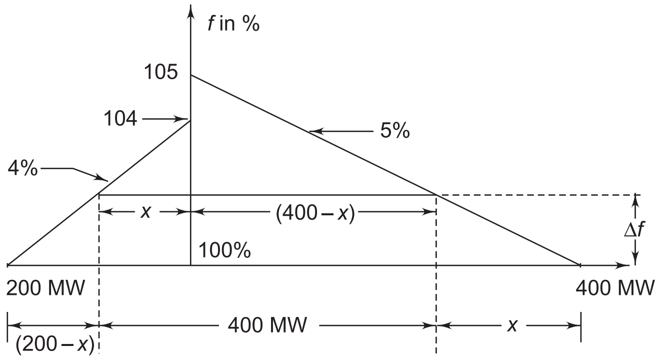


Fig. S-8.1 (a)

Generator 1 = 200 MW, 4% droop

2 = 400 MW, 5% droop

As the load is reduced to 400 MW, let

load on gen 1 = x MW

∴ load on gen 2 = $(400 - x)$ MW

Rise in freq. = Δf

Now $\Delta f/(200 - x) = 0.04 \times 50/200$ (i)

$\Delta f/x = 0.05 \times 50/400$ (ii)

Equating Δf in Eqs (i) and (ii), we get

x = **123** MW (load on gen 1)

∴ $400 - x$ = **277** MW (load on gen 2)

$$\text{System freq.} = 50 + \frac{0.05 \times 50}{400} \times 123 = \mathbf{50.77}\ \text{Hz}$$

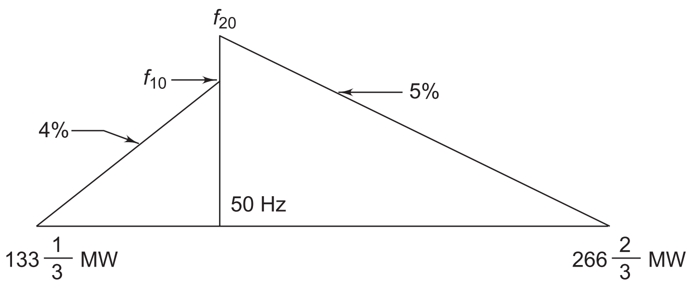


Fig. S-8.1 (b)

$$\frac{f_{10} - 50}{400/3} = \frac{0.04 \times 50}{200} \text{ or } f_{10} = \mathbf{51\frac{1}{3}}\ \text{Hz}$$

$$\frac{f_{20} - 50}{800/3} = \frac{0.05 \times 50}{400} \quad \text{or} \quad f_{20} = \mathbf{51\frac{2}{3}} \text{ Hz}$$

8.2

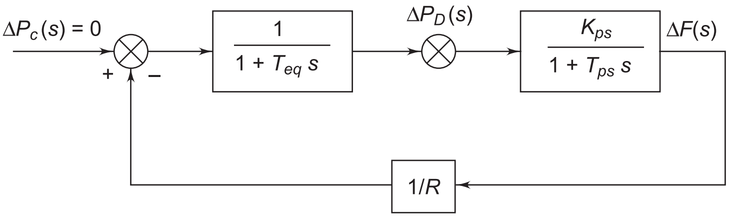


Fig. S-8.2

$$K_{sg}\ K_t = 1$$

$$\frac{1}{1+T_{eq}s} = \frac{1}{1+0.9s};\ \frac{K_{ps}}{1+T_{ps}s} = \frac{100}{1+20s};\ \frac{1}{R} = \frac{1}{3}$$

$$\Delta F(s) = \frac{100/(1+20s)}{1+\dfrac{100}{1+20s}\times\dfrac{1}{3}\times\dfrac{1}{1+0.9s}} \times \frac{0.01}{s}$$

$$= \frac{0.056\,(1+0.9s)}{s(s^2+1.16s+1.91)}$$

$$s = \left(-1.16 \pm \sqrt{(1.16)^2 - 7.64}\right)\Big/2 = -0.58 \pm j\ 1.254$$

$$\Delta F(s) = -\frac{0.056\,(1+0.9s)}{s(s+0.58+j1.254)\,(s+0.58-j1.254)}$$

$$\Delta f(t) = -0.029 - 2\ \text{Re}\left\{\frac{0.056\,(1+0.9s)}{s(s+0.58-j1.254)}\bigg|_{s=-(0.58+j1.254)} e^{-(0.58+j1.254)t}\right\}$$

$$= \mathbf{-0.029 - 0.04\ e^{-0.58t}\cos(1.254\ t + 137.8°)}$$

$\Delta f(t)$ vs t can be plotted from this equation. The response has now become oscillatory and is therefore closer to the exact response.

8.3 $$\Delta F(s) = -\frac{\dfrac{K_{ps}}{(1+T_{ps}s)}}{1+\dfrac{1}{(1+T_{sg}s)(1+T_t s)}\times\left(\dfrac{1}{R}+\dfrac{K_i}{s}\right)\left(\dfrac{K_{ps}}{1+s\,T_{ps}}\right)} \times \frac{1}{s}$$

$$\int_{0}^{t} \Delta f(t)\ dt \Bigg|_{\lim t\to\infty} = \lim_{s\to 0} \Delta F(s) = \frac{1}{K_i} \text{ cycles} = \frac{1}{K_i}\times\frac{1}{50} \text{ sec.}$$

Error in cycles is inversely proportional to K_i, the gain of integral controller.

8.4 Due to integral action of the block $\frac{2\pi T_{12}}{s}$, $[\Delta f_1(t) - \Delta f_2(t)]$ would go to zero as $t \to \infty$ i.e. $\Delta f_1\ (\infty) = \Delta f_2\ (\infty) = \Delta f$

Under steady condition substitute $s = 0$ for all other blocks (these are time constants).

For area 1

$$\Delta f = \left\{-\ (b_1\ \Delta f + \Delta\ P_{\text{tie, 1}})\ K_{i1} - \frac{1}{R_1}\ \Delta f - \Delta \text{P}_{\text{tie, 1}} - 1\right\}\ K_{ps1}$$

For area 2

$$\Delta f = \left\{-\ (b_2\ \Delta f - a_{12}\ \Delta P_{\text{tie, 1}})\ K_{i2} - \frac{1}{R_1}\ \Delta f + a_{12}\ \Delta P_{\text{tie, 1}} - 1\right\}\ K_{ps2}$$

Reorganising we get

$$\left(\frac{1}{K_{p\,s1}} + K_{i1}\ b_1 + \frac{1}{R_1}\right)\ \Delta f + (K_{i1} + 1)\ \Delta P_{\text{tie,1}} = -\ 1$$

$$\left(\frac{1}{K_{p\,s2}} + K_{i2}\ b_2 + \frac{1}{R_2}\right)\ \Delta f - a_{12}\ (K_{i2} + 1)\ \Delta P_{\text{tie, 1}} = -\ 1$$

Solving we get

$$\Delta f = -\frac{a_{12}\ (K_{i2} + 1) + (K_{i1} + 1)}{a_{12}\ (K_{i2} + 1)\left(\frac{1}{K_{ps1}} + K_{i1}\, b_1 + \frac{1}{R_1}\right) + (K_{i1} + 1)\left(\frac{1}{K_{ps2}} + K_{i2}\, b_2 + \frac{1}{R_2}\right)}$$

$\Delta P_{\text{tie, 1}} =$

$$\frac{\left(\frac{1}{K_{ps}} + K_{i1}\ b_1 + \frac{1}{R_1}\right) - \left(\frac{1}{K_{ps2} + K_{i2}\ b_2 + \frac{1}{R_2}}\right)}{a_{12}\ (K_{i2} + 1)\left(\frac{1}{K_{ps1}} + K_{i1}\ b_1 + \frac{1}{R_1}\right) + (K_{i1} + 1)\left(\frac{1}{K_{ps2}} + K_{i2}\ b_2 + \frac{1}{R_2}\right)}$$

8.5. For area 1

$$-\ [\Delta P_{\text{tie, 1}}\ (s) + b\ \Delta F_1\ (s)] \times \frac{K_i\ K_{ps}}{s\,(1 + T_{sg}\,s)\,(1 + T_t s)\,(1 + T_{ps} s)}$$

$$-\frac{1}{R} \times \frac{K_{ps}}{(1 + T_{gs} s)\,(1 + T_t s)\,(1 + T_{ps} s)}\ \Delta F_1\ (s) - \frac{K_{ps}}{(1 + T_{ps} s)}\ \Delta P_{\text{tie, 1}}(s)$$

$$-\ \frac{K_{ps}}{(1 + T_{ps} s)}\ \Delta P_{D1}\ (s) = \Delta F_1\ (s)$$

$$\left[1+\frac{42.5}{s(1+0.4s)(1+0.5s)(1+20s)}+\frac{33.3}{(1+0.4s)(1+0.5s)(1+20s)}\right]\Delta F_1\ (s)$$

$$+\left[\frac{42.5}{s(1+0.4s)(1+0.5s)(1+20s)}+\frac{100}{(1+20s)}\right]\Delta P_{\text{tie, 1}}\ (s) = -\frac{100}{(1+20s)}\times\frac{1}{s}$$

$[s\ (1 + 0.4\ s)\ (1 + 0.5\ s)\ (1 + 20\ s) + 42.5 + 33.3\ s]\ \Delta F_1\ (s)$
$+ [42.5 + 100\ s\ (1 + 0.4\ s)\ (1 + 0.5\ s]\ \Delta P_{\text{tie, 1}}\ (s) = -100$
$(1 + 0.4\ s)\ (1 + 0.5\ s)$ (i)

For area 2

$[s\ (1 + 0.4\ s)\ (1 + 0.5\ s)\ (1 + 20\ s) + 42.5 + 33.3\ s_1]\ \Delta F_2$ (s)
$-[42.5 + 100\ s\ (1 + 0.4\ s)\ (1 + 0.5\ s)]\ \Delta P_{\text{tie, 1}}\ (s) = 0$

$$[\Delta F_1\ (s) - \Delta F_2\ (s)] \times \frac{2\pi T_{12}}{s} = \Delta P_{\text{tie, 1}}\ (s) = 0 \quad \text{(ii)}$$

$$\Delta F_1\ (s) = \Delta F_2\ (s) + 20s\ \Delta P_{\text{tie, 1}}\ (s) \quad \text{(iii)}$$

$(4s^4 + 18.2s^3 + 20.9s^2 + 34.3s + 42.5)\ \Delta F_1(s) + (20s^3 + 90s^2$
$+ 100s + 42.5)\ \Delta P_{\text{tie, 1}}\ (s) = -100\ (0.2\ s^2 + 0.9\ s + 1)$ (iv)

$(4s^4 + 18.2\ s^3 + 20.9\ s^2 + 34.3\ s + 42.5)\ \Delta f_2\ (s) - (20s^3 + 90s_2$
$+ 100s + 42.5)\ \Delta P_{\text{tie, 1}}\ (s) = 0$ (v)

$$\Delta F_1\ (s) = \Delta F_2\ (s) + 20s\ \Delta P_{\text{tie, 1}}\ (s) \quad \text{(vi)}$$

$(4s^4 + 18.2s^3 + 20.9s^2 + 34.3s + 42.5)\ \Delta F_2\ (s) + (80s^5 + 364s^4$
$+ 438s^3 + 776s^2 + 950s + 42.5)\ \Delta P_{\text{tie, 1}}\ (s)$
$= -100\ (0.2\ s^2 + 0.9s + 1)$

$(4s^4 + 18.2s^3 + 20.9s^2 + 34.3s + 42.5)\ \Delta F_2(s) - (20s^3 + 90s^2$
$+ 100s + 42.5)\ \Delta P_{\text{tie, 1}}\ (s) = 0$

From which we get

$$\Delta P_{\text{tie, 1}}\ (s) = -\frac{100\,(0.2\,s^2 + 0.9\,s + 1)}{80\,s^5 + 364\,s^4 + 458\,s^3 + 866\,s^2 + 1050\,s + 85}$$

To check stability apply Routh's criterion to the characteristic equation

$80s^5 + 364s^4 + 458s^3 + 866s^2 + 1{,}050s + 85 = 0$

s^5	80	458	1,050
s^4	364	866	85
s^3	267.7	1031	
s^2	– 536.9		
s^1			
s^0			

Clearly, the system is found to be *unstable*.

Chapter 9

9.1 $Z = 5 + j\,314 \times 0.1 = 5 + j\,31.4 = 31.8\ \angle 81°$; $L/R = \dfrac{0.1}{5} = 0.02$ sec.

Substituting in Eq. (9.1)

$$i_z = \frac{100}{31.8}\sin(314t + 15° - 81°) + \frac{100}{31.8}\sin(81° - 15°)e^{-50t}$$

$$= \mathbf{3.14\ sin\ (314\ \mathit{t} - 66°) + 2.87\mathit{e}^{-50\mathit{t}}}$$

First current maximum of symmetrical s.c. current occurs at

$57.3 \times 314t - 66° = 90°$; $\therefore\ t = 0.00867$ sec

First current maximum

$$i_{mm} = 3.14 + 2.87\ e^{-50 \times 0.00867} = \mathbf{5}\ \text{A}$$

9.2 For dc off-set current to be zero: $\alpha - \theta =$ **81°**

(b) For dc offset current to be maximum: $\theta - \alpha = 90°\ \therefore\ \alpha = \theta - 90° =$ **–9°**

9.3

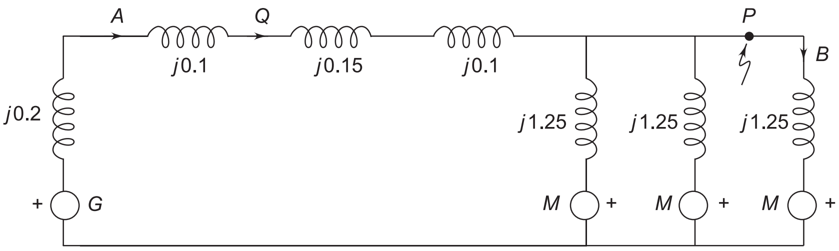


Fig. S-9.3

All voltages before fault are 1 pu as the system is operating on no load.

(i) Fault at P

Current to be interrupted by circuit breaker A,

$$I_A = \frac{-j}{0.2 + 0.1 + 0.15 + 0.1}$$

(Base current in gen circuit $= 25/\sqrt{3} \times 11 = 1.312$ kA) $I_A = -j\,1.818$ pu

$\therefore$ $I_A =$ **2.386** kA

Current to be interrupted by circuit breaker B,

$$I_B = \frac{1}{j1.25} = -\,j0.8 \text{ pu}$$

Base current in motor circuit $= 25/\sqrt{3} \times 6.6 = 2.187$ kA

$\therefore$ $I_B =$ **1.75** kA

(ii) Fault at Q

$$I_A = \frac{-j}{0.2 + 0.1} = -j\ 3.33 \text{ pu} = \mathbf{4.373} \text{ kA}$$

$$I_B = \frac{1}{j1.25} = -j\ 0.8 \text{ pu} = \mathbf{1.75} \text{ kA}$$

9.4

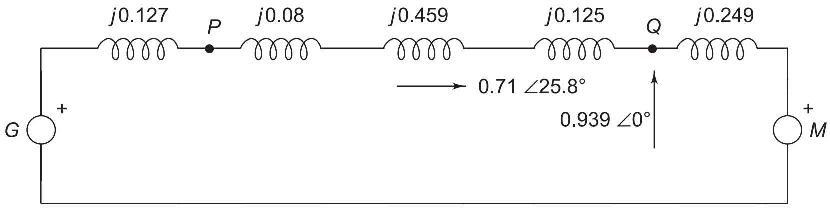


Fig. S-9.4

Base MVA = 25; Voltage base in gen circuit = 11 kV
voltage base in line circuit = 33 kV
voltage base in motor circuit = 3.3 kV

Calculation of pu reactances

$$\text{Gen} = 0.1 \times (12.4/11)^2 = 0.127$$

$$\text{Motor} = 0.15 \times (25/20) \times (3.8/3.3)^2 = 0.249$$

$$\text{Line} = 20 \times 25/(33)^2 = 0.459;$$

Transformer $T_1 = 0.08$

Transformer $T_2 = 0.1 \times 25/20 = 0.125;$

Motor Load: $\frac{15}{25} = 0.6$ MW (Pu) pf 0.9 leading or $\angle 25.8°$

Terminal voltage = 3.1/3.3 = 0.939 pu

Motor current = $0.6/(0.939 \times 0.9) = 0.71 \angle 25.8°$ pu

Under conditions of steady load:

Voltage at generator terminals

$$= 0.939 \angle 0° + 0.71 \angle 25.8° (0.08 + 0.459 + 0.125) \angle 90°$$

$$= 0.734 + j\ 0.424 = 0.847 \angle 30°$$

Thévenin equivalent voltage as seen from P: $V° = 0.847 \angle 30°$

Current caused by fault in gen circuit (towards P) = $\frac{0.847 \angle 30°}{j0.127} = 6.67$ $\angle -60°$

$$(I_B \text{ (Gen)} = 25/(\sqrt{3} \times 11) = 1.312 \text{ kA};$$

$$I_B \text{ (Motor)} = 25/(\sqrt{3} \times 3.3) = 4.374 \text{ kA})$$

Current caused by fault in motor circuit (towards P) $= \dfrac{0.847 \angle 30°}{j\,0.913}$

$= 0.93 \angle -60°$

Motor current during fault $= -0.71 \angle 25.8° + 0.93 \angle -60°$

$= -0.174 - j\,1.114$ pu = **4.93** kA

9.5 Base: 1 MVA, 0.44 kV; Line reactance $= \dfrac{0.05 \times 1}{(0.44)^2} = 0.258$ pu

Reactance of large system = 1/8 = 0.125 pu

Operating voltage at motor bus before fault $= \dfrac{0.4}{0.44} = 0.909$ pu

Short circuit current fed to fault at motor bus $= 0.909 \left(\dfrac{1}{0.125 + 0.258} + 2 \times \dfrac{1}{0.1} \right) = 20.55$ pu

Base current $= 1/(\sqrt{3} \times 0.44) = 1.312$ kA

∴ Short circuit current = **26.96** kA

9.6 Base: 0.5 MVA, 0.44 kV,

Base current $= \dfrac{0.5}{\sqrt{3} \times 0.44} = 0.656$ kA

Load $= \dfrac{0.4}{0.5} = 0.8$ MW (pu)

$pf = 0.8$ lagging or $\angle -36.9°$

Load current before fault $= \dfrac{0.8}{0.8} \angle -36.9° = 1 \angle -36.9°$ pu

Thévenin voltage $V° = 1 \angle 0°$ pu; Thévenin, reactance $= j = 0.1$ pu

Gen current caused by fault $= \dfrac{1}{j\,0.1} = -j\,10$ pu

Post fault current at gen terminals $= -j\,10 + 1 \angle -36.9° = 0.8 - j\,10.6$

$= 10.63 \angle -85.7°$ = **6.97** kA

9.7 Bus: 10 MVA, 6.6 kV (Gen), 6.6/31.56 kV (transformer)

Base current $= 10/(\sqrt{3} \times 31.56) = 0.183$ kA

Gen reactances: $x''_d = 0.1$, $x'_d = 0.2$, $x_d = 0.8$ pu

Transformer reactance: $0.08 \times (6.9/6.6)^2 = 0.0874$ pu

No load voltage before fault = 30/31.56 = 0.95 pu

(a) Initial symmetrical rms current = $\dfrac{0.95}{0.1 + 0.0874}$ = 5.069 pu

= **0.9277** kA

(b) Max. possible dc off-set current = $\sqrt{2}$ × 0.9277 = **1.312** kA

(c) Momentary current (rms) rating of the breaker = 1.6 × 0.9277

= **1.4843** kA

(d) Current to be interrupted by the breaker (5 cycle) = 1.1 × 0.9277

= **1.0205** kA; Interrupting MVA = $\sqrt{3}$ × 30 × 1.0205 = **53.03** MVA

(e) Sustained short circuit current in breaker = $\dfrac{0.95}{0.8 + 0.0874}$ × 0.183

= **0.1959** k/a

9.8

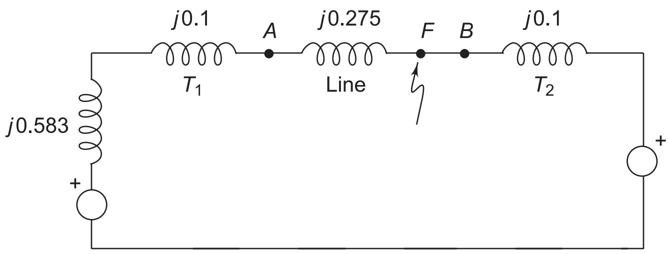


Fig. S-9.8

Base: 100 MVA; 12 kV (Gen. ckt), 66 kV (line)

Base current (gen ckt) = $100/(\sqrt{3} \times 12)$ = 4.81 k/A

Base current (line) = $100/(\sqrt{3} \times 66)$ = 0.875 kA

Component reactances

Gen: 0.35 × (100/60) = 0.583 pu;

Line: = $\dfrac{12 \times 100}{(66)^2}$ = 0.275 pu

Transformer: 0.08 × (100/80) = 0.1 pu

Load: 50/100 = 0.5 pu,

11/12 = 0.917 pu,

pf = 0.8 lag; ∠– 36.9°

Load current = 0.5/0.917 = 0.545 pu

Thévenin voltage at *F* before fault, $V°$ = 0.917 ∠0°

Current through breaker A due to fault $= \dfrac{0.917}{j(0.583 + 0.1 + 0.275)}$

$= 0.957 \angle -90°$

Post fault current through breaker $A = 0.957 \angle -90° + 0.545 \angle -36.9°$

$= 0.436 - j\,1.284 = 1.356$ pu $= 6.522$ kA

Current through breaker B due to fault $= 0.917/j\,0.1 = 9.17 \angle -90°$

Post fault current through breaker $B = 9.17 \angle -90° + 0.545 \angle -36.9°$

$= 0.436 - j\,9.497 = 9.507$ pu

$=$ **8.319** kA

9.9

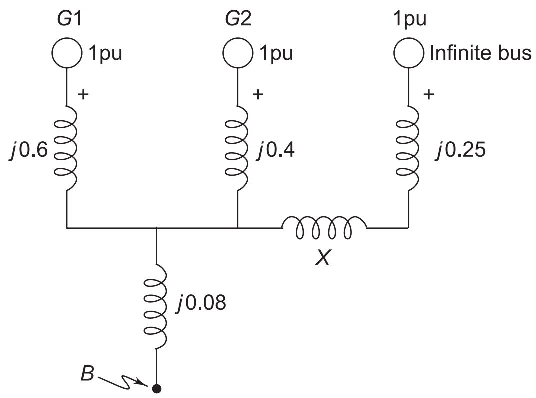


Fig. S-9.9

Assumption: All reactances are given on appropriate voltage bases. Prefault no load voltage = 1pu

Base: 100 MVA

SC rupturing capacity of breaker $= \dfrac{333}{100} = 3.33$ pu

Equivalent system reactance = 1/3.33 = 0.3 pu

Equivalent system reactance at gen bus = 0.3 – 0.08 = 0.22 pu

Now

$$\frac{1}{0.22} = \frac{1}{0.6} + \frac{1}{0.4} + \frac{1}{x + 0.25} \quad \therefore\ X = \mathbf{2.39} \text{ pu}$$

9.10

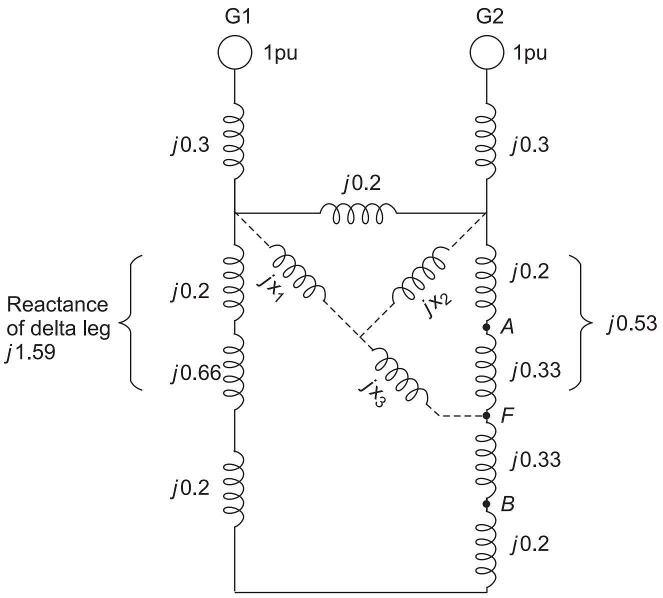


Fig. S-9.10

Base: 100 MVA, 110 kV on lines

Component reactances (pu):

$$G_1 = 0.3$$

$$G_2 = 0.18 \times \frac{100}{60} = 0.3$$

Transformer (each): 0.2

Inductive reactor: 0.2

Line (each): $\frac{80 \times 100}{(110)^2} = 0.66$

Converting delta to star (shown dotted)

$$X_1 = \frac{1.59 \times 0.2}{2.32} = 0.137;$$

$$X_2 = \frac{0.2 \times 0.53}{2.32} = 0.046$$

$$X_3 = \frac{0.53 \times 1.59}{2.32} = 0.363$$

Equivalent reactance $= (0.3 + 0.137) \parallel (0.3 + 0.046) + 0.363 = 0.556$

Fault current $I^f = \frac{1}{j\,0.556} = -\,j\;1.8$

Let us now determine the part of I^f that flows through A and the part that flows through B

$$I^f_{G1} = -j\ 1.8 \times \frac{0.346}{0.783} = -j\ 0.795$$

$$I^f_{G2} = -j\ 1.8 \times \frac{0.437}{0.783} = -j\ 1.005$$

$$V_2 = 1 - (-j\ 1.005) \times j\ 0.3 = 0.6988 \approx 0.7$$

$$V_1 = 1 - (-j\ 0.795) \times j\ 0.3 = 0.7615 \approx 0.762$$

$$I^f_A = 0.7/j\ 0.53 = -j\ 1.321$$

SC MVA through $A = 1.321 \times 100 =$ **132.1**

$$I^f_B = 0.762/j1.59 = -j\ 0.479$$

SC MVA through $B = 0.479 \times 100 =$ **47.9**

If reactor X is eliminated

Equivalent reactance $= (0.3\ //\ 0.3) + (1.59\ //\ 0.53) = j\ 0.5475$

$$I^f = -j\ 1.826$$

$$I^f_A = j\ 1.826 \times \frac{1.59}{2.12} = -j\ 1.369 \text{ SC MVA} = \mathbf{136.9}$$

$$I^f_B = -j\ 1.826 \times \frac{0.53}{2.12} = -j\ 0.456 \text{ SC MVA} = \mathbf{45.6}$$

There is no significant change in SC MVA through A and B caused by X.

9.11

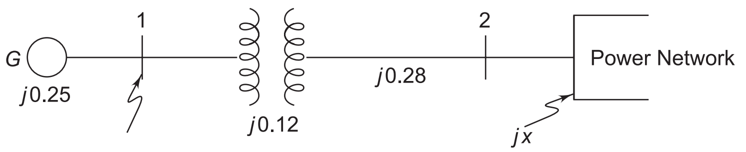


Fig. S-9.11

Fault current contribution by generator $= \dfrac{1}{j\,0.25} = -j\ 4$

Fault current contribution by power network

$$= -j\ 1 = \frac{1}{j\,0.12 + j\,0.28 + j\,X}$$

$\therefore \quad X + 0.4 = 1$

$\therefore \quad X =$ **0.6** pu

9.12 From the network of Fig. P-9.12, we can write

$$Y_{\text{BUS}} = \begin{bmatrix} -j\,26.67 & j\,10 & j\,10 \\ j\,10 & -j\,26.67 & j\,10 \\ j\,10 & j\,10 & -j\,20 \end{bmatrix}$$

Inverting,

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} j0.0885 & j0.0613 & j0.0749 \\ j0.0613 & j0.0885 & j0.0749 \\ j0.0749 & j0.0749 & j0.1249 \end{bmatrix}$$

Using Eq. (9.26), $V_1^f = V_1^0 - (Z_{13}/Z_{23})\ V_3^0$

The prefault condition being no load, $V_1^0 = V_2^0 = V_3^0 = 1$ pu

$$\therefore \qquad V_1^f = 1.0 - \frac{j0.0749}{j0.1249} \times 1 = 0.4004 \text{ pu } //^{ly}$$

$$V_2^f = 0.4004; \; V_3^f = 0$$

From Eq. (9.25) $\quad I_f = 1.0/j\ 0.1249 = -\ j\ \mathbf{8.006}$ pu

S.C. current in line 1–3

$$I_{13}^f = \frac{V_1^f - V_3^f}{z_{13}} = \frac{0.4004 - 0}{j0.1} = -\ j\ \mathbf{4.094} \text{ pu}$$

The fault current for a fault on bus 1 (or bus 2) will be

$$I^f = \frac{1.00}{Z_{11}\,(\text{or } Z_{22})}$$

$$= \frac{1.00}{j0.0885} = -\ j\ 11.299 \text{ pu.}$$

Chapter 10

10.1

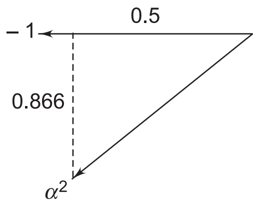


Fig. S-10.1

(i) $\alpha^2 - 1 = -1.5 - j\,0.866 = \mathbf{1.732\ \angle 210°}$

(ii) $1 - \alpha - \alpha^2 = 1 - (-0.5 + j\,0.866) - (-0.5 - j\,0.866) = \mathbf{2\angle 0°}$

(iii) $3\alpha^2 + 4\alpha + 2 = 3\,(-0.5 - j\,0.866) + 4\,(-0.5 + j\,0.866) + 2$
$= \mathbf{1.732\ \angle 150°}$

(iv) $j\alpha = 1\ \angle 90° \times 1\ \angle 120° = 1\ \angle 210°$

10.2

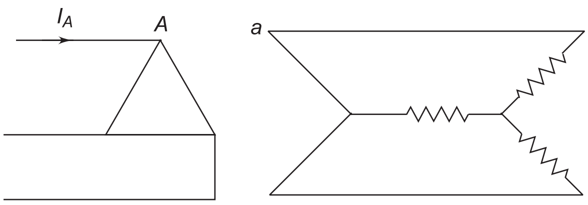


Fig. S-10.2 a

Base: 750 kVA, 2,500 V; Load: 1 pu kVA, 1 pu V

Load voltages $|V_{ab}| = 0.8,\ |V_{bc}| = 1.16,\ |V_{ca}| = 1.0$

Choosing phase angle of V_{bc} to be – 90°

$$(0.8)^2 = (1.16)^2 + (1)^2 - 2 \times 1.16 \cos\ \theta$$

$$\therefore \qquad \theta = 42.7°$$

$$V_{ca} = 1.0\ \angle 132.7°$$

$$(1.16)^2 = (1)^2 + (0.8)^2 - 2 \times 0.8 \cos\ \gamma$$

$$\gamma = 79.4° \therefore \mathrm{V}_{ab} = 0.8\ \angle\ 32.1°\ ;\ \mathrm{V}_{bc} = 1.16\ \angle -90°$$

$$V_{ab1} = \frac{1}{3}\ [0.8\ \angle 32.1° + 1.16\ \angle 30° + 1\ \angle 12.7°]$$

$$= 0.975\ \angle 24.7°$$

$$V_{ab2} = \frac{1}{3}\ [0.8\ \angle 32.1° + 1.16\ \angle 150° + 1\ \angle -107.3°]$$

$$= 0.21\ \angle 175.3°$$

(line-to-line voltage base)

$$V_{a1} = V_{ab1}\ \angle -30° = 0.975\ \angle -5.3°$$

(line-to-neutral voltage base)

$$V_{a2} = V_{ab2} \angle 30° = 0.21 \angle 205.3°$$

(line-to-neutral voltage base)

Assuming + 90° connection

$$V_{A1} = V_{a1} \angle 90° = 0.975 \angle 84.7°;$$

$$V_{A2} = V_{a2} \angle -90° = 0.21 \angle 115.3°$$

Load resistance = 1 pu to both positive and negative sequence currents.

$$\therefore \quad I_{A1} = 0.975 \angle 84.7° \text{ pu;}$$

$$I_{A2} = 0.21 \angle 115.3° \text{ pu}$$

$$I_A = I_{A1} + I_{A2}$$

$$= 0.975 \angle 84.7° + 0.21 \angle 115.3°$$

$$= 0.0003 + j1.16 = \mathbf{1.16 \angle 90°} \text{ pu}$$

Similarly I_B and I_C can be found.

$$V_{AB1} = V_{A1} \angle 30° = 0.975 \angle 114.7°$$

$$V_{AB2} = V_{A2} \angle -30° = 0.21 \angle 85.3°$$

$$V_{AB} = V_{AB1} + V_{AB2} = 0.975 \angle 114.7° + 0.21 \angle 85.3°$$

$$= \mathbf{1.17 \angle 109.5°} \text{ pu}$$

$$V_{BC} = \alpha^2 V_{AB1} + \alpha V_{AB2}$$

$$= 0.975 \angle -53° + 0.21 \angle -154.7°$$

$$= \mathbf{0.953 \angle -65.4°} \text{ pu}$$

$$V_{CB} = \alpha V_{AB1} + \alpha^2 V_{AB2}$$

$$= 0.975 \angle -125.3° + 0.21 \angle -34.7°$$

$$= \mathbf{0.995 \angle -113.1°} \text{ pu}$$

10.3

$$V_{a1} = \frac{1}{3} [200 + 200 \angle 5° + 200 \angle -15°]$$

$$= \mathbf{197.8 \angle -3.3°} \text{ V}$$

$$V_{a2} = \frac{1}{3} [200 + 200 \angle 125° + 200 \angle -135°]$$

$$= \mathbf{20.2 \angle 158.1°} \text{ V}$$

$$V_{a0} = \frac{1}{3} [200 + 200 \angle 245° + 200 \angle 105°]$$

$$= 21.61 \angle 10.63° \text{ V}$$

10.4

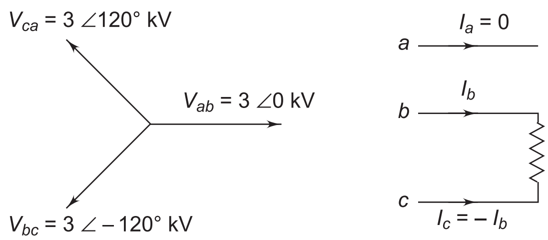


Fig. S-10.4

$$I_b = \frac{100}{3} \angle -120° = 33.3 \angle -120° \ A;$$

$$I_c = 33.3 \angle 60° \ A; \ I_a = 0$$

$$I_{a0} = \mathbf{0}$$

$$I_{a1} = \frac{1}{3} [33.3 + 33.3 \angle -60°] = \mathbf{19.23 \angle -30°} \text{ A}$$

$$I_{a2} = \frac{1}{3} [33.3 \angle 120° + 33.3 \angle 180°]$$

$$= \mathbf{19.23 \angle 150°} \text{ A}$$

10.5

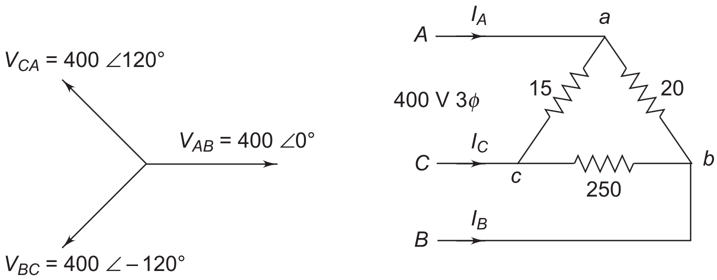


Fig. S-10.5

$$I_{ab} = \frac{400}{20} = 20 \angle 0° \ A$$

$$I_{bc} = \frac{400}{250} \angle -120° = 1.6 \angle -120° \ A$$

$$I_{ca} = \frac{400}{15} \angle 120° = 26.7 \angle 120° \ A$$

$$I_A = I_{ab} - I_{ca} = 20 - 26.7 \angle 120°$$

$$= 40.58 \angle -34.7°$$

$$I_B = I_{bc} - I_{ab} = 1.6 \angle -120° - 20$$

$$= 20.84 \angle 183.8°$$

$$I_c = I_{ca} - I_{bc} = 26.7 \angle 120° - 1.6 \angle -120°$$

$$= 27.54 \angle 117.1°$$

$$I_{A1} = \frac{1}{3} [40.58 \angle -34.7° + 20.84 \angle -56.2° + 27.54 \angle -2.9°]$$

$$= \mathbf{27.87 \angle -30°}$$

$$I_{A2} = \frac{1}{3} [40.58 \angle -34.7° + 20.84 \angle 63.8° + 27.54 \angle -122.9°]$$

$$= \mathbf{13 \angle -44.93°}$$

$$I_{A0} = \mathbf{0}$$

$$I_{ab1} = \frac{1}{3}\ [I_{ab} + \alpha I_{bc} + a^2 I_{ca}]$$

$$= \frac{1}{3}\ [20 + 1.6 + 26.7] = \mathbf{16.1}\ A$$

$$I_{ab2} = \frac{1}{3}\ [20 + 1.6\ \angle 120° + 26.7\ \angle 240°]$$

$$= \mathbf{7.5\ \angle{-}\ 74.94°}\ A$$

$$I_{ab0} = \frac{1}{3}\ [20 + 1.6\ \angle{-}\ 120° + 26.7\ \angle 120°]$$

$$= \mathbf{7.5\ \angle 74.94°}$$

10.6 Obviously $I_{a1} = 0$

$$V_{a1} = Z_{11}\ I_{a1} + Z_{12}\ I_{a2} \qquad \text{(i)}$$

$$V_{a2} = Z_{21}\ I_{a1} + Z_{22}\ I_{a2} \qquad \text{(ii)}$$

Now $V_{a1} = 200\ \angle 0°;\ V_{a2} = 0$

(a balanced 3ϕ supply is assumed)

$$Z_{11} = \frac{1}{3}\ (10 + 15 + 20) = 15\ \angle 0°$$

$$Z_{12} = \frac{1}{3}\ (10 + 15\ \angle{-}\ 120° + 20\ \angle 120°)$$

$$= -\ 2.5\ j\ 1.44 = 2.89\ \angle 150°$$

$$Z_{21} = \frac{1}{3}\ (10 + 15\ \angle 120° + 20\ \angle{-}\ 120°)$$

$$= -\ 2.5 - j\ 1.44 = 2.89\ \angle{-}\ 150°$$

$$Z_{22} = \frac{1}{3}\ (10 + 15 + 20) = 15\ \angle 0°$$

Substituting in (i) and (ii) we get

$$200 = 15\ I_{a1} + 2.89\ \angle 150°\ I_{a2} \qquad \text{(iii)}$$

$$0 = 2.89\ \angle{-}\ 150°\ I_{a1} + 15\ I_{a2} \qquad \text{(iv)}$$

Solving (iii) and (iv) for I_{a1} and I_{a2}, we have

$$I_{a2} = 2.67\ \angle 30°;\ I_{a1} = 13.85\ \angle 0°$$

Currents in load branches

$$I_a = 13.85 + 2.67\ \angle 30° = \mathbf{16.16 + j\ 1.335}\ A$$

$$I_b = 13.85\ \angle{-}\ 120° + 2.67\ \angle 150° = -\ \mathbf{9.24 - j\ 10.66}\ A$$

$$I_c = 13.85\ \angle 120° + 2.67\ \angle{-}\ 90° = -\ \mathbf{6.93 + j\ 9.32}\ A$$

$$V_{a0} = Z_{01}\ I_{a1} + Z_{02}\ I_{a2}$$

From Eq. (10.40)

$$Z_{01} = Z_{12} = 2.89\ \angle 150°$$

$$Z_{02} = Z_{21} = 2.89\ \angle{-}\ 150°$$

$\therefore \quad V_{a0} = 2.89 \angle 150° \times 13.85 \angle 0° + 2.89 \angle -150° \times 2.67 \angle 30°$

$= 40.75 \angle 160.9°$

$|V_{Nn}| = |V_{a0}| =$ **40.75** volts

10.7

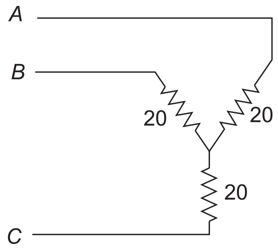


Fig. S-10.7

$V_{AB} = 200 \angle 0°, \; V_{BC} = 100 \angle 255.5°,$

$V_{CA} = 200 \angle 151°$

Check $V_{AB} + V_{BC} + V_{CA} = 0$

$V_{AB1} = \frac{1}{3} [200 + 100 \angle 15.5° + 200 \angle 31°]$

$= \mathbf{161.8 \angle 15.5°}$

$V_{AB2} = \frac{1}{3} [200 + 100 \angle 135.5° + 200 \angle -89°]$

$= 61.8 \angle -44.5°$

$V_{A1} = \frac{161.8}{\sqrt{3}} \angle -14.5° = \mathbf{93.4 \angle -14.5°}$

$V_{A2} = \frac{61.8}{\sqrt{3}} \angle -14.5° = \mathbf{35.7 \angle -14.5°}$

$I_{A1} = \frac{93.4}{20} = 4.67 \angle -14.5°;$

$I_{A2} = \frac{35.7}{20} = 1.79 \angle -14.5°$

$I_A = 4.67 \angle -14.5° + 1.79 \angle -14.5°$

$= \mathbf{6.46 \angle -14.5°}$

$I_B = 4.67 \angle 225.5° + 1.79 \angle 105.5°$

$= \mathbf{4.08 \angle -156.8°}$

$I_C = 4.67 \angle 105.5° + 1.79 \angle 225.5°$

$= \mathbf{4.08 \angle 127.8°}$

Positive sequence power = $3 \times 93.4 \times 4.67 = 1308.5$

Negative sequence power = $3 \times 35.7 \times 1.79 =$ **191.7**

Total power **1,500.2** Watts

Check $P = 20\,(6.46^2 + 4.08^2 + 4.08^2) = 1{,}500.4$ Watts.

10.8 Base: 50 MVA, 220 kV (in line), 11 kV (Gen. 1 and 2)

$$X''_{g1} = 0.2 \times \frac{50}{25} = 0.4 \qquad X_0 \text{ (each m/c)} = 0.08 \times \frac{50}{25} = 0.16$$

$$X''_{g2} = 0.4, \qquad X_T \text{ (each)} = 0.15 \times \frac{50}{25} = 0.375$$

$$X_L = \frac{50 \times 50}{(220)^2} = 0.052 \qquad X_{L0} = 0.052 \times 2.5 = 0.13$$

$$\text{Grounding reactance (each)} = 0.05 \times \frac{50}{25} = 0.1$$

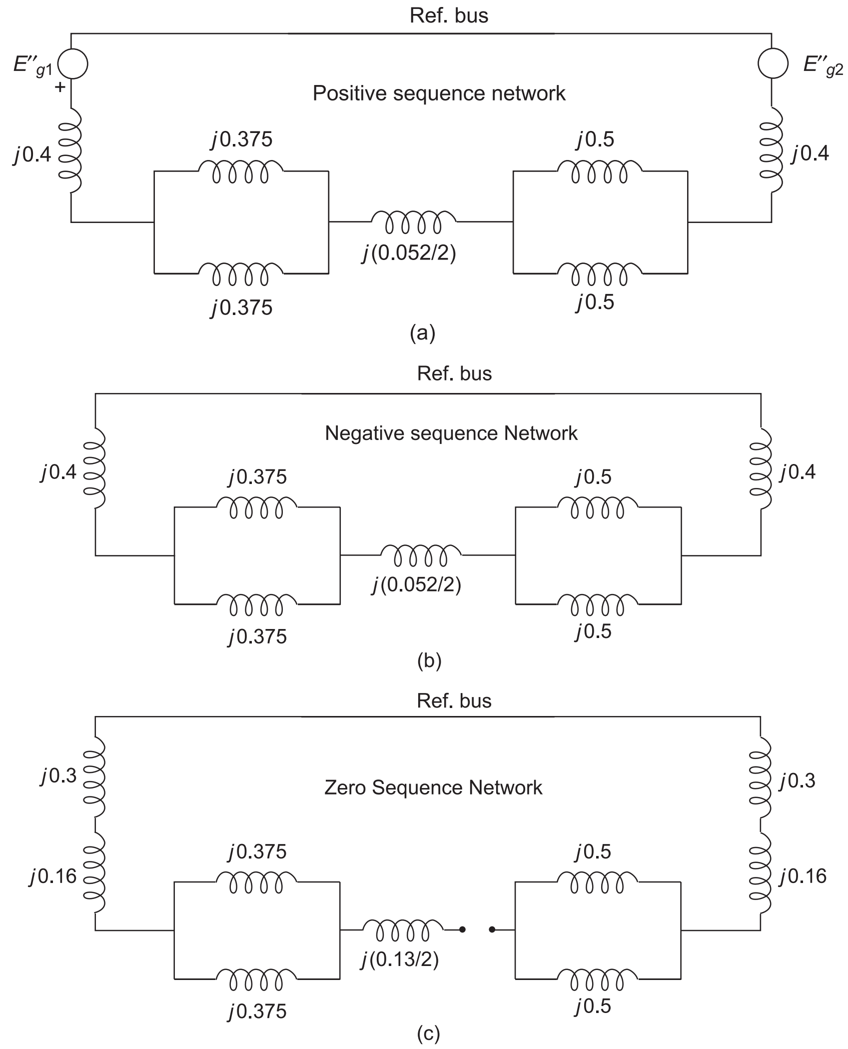


Fig. S-10.8

10.9 Base: 50 MVA, 11 kV (Gen 1, 2, Motor), 120 kV (line)

Gen 1: $X'' = 0.4$, $X_2 = 0.3$, $X_0 = 0.06$

Gen 2: $X'' = 0.67$, $X_2 = 0.5$, $X_0 = 0.17$

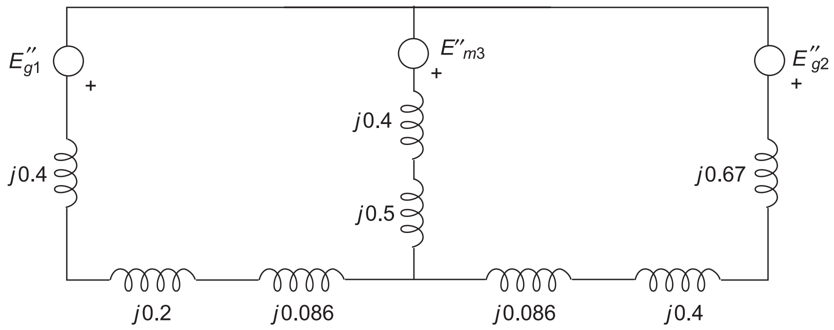
Mot. 3: $X'' = 0.4$, $X_2 = 0.4$, $X_0 = 0.2$

Transf. 1: $X = 0.2$, Transf. 2: $X = 0.4$, Transf. 3: $X = 0.5$

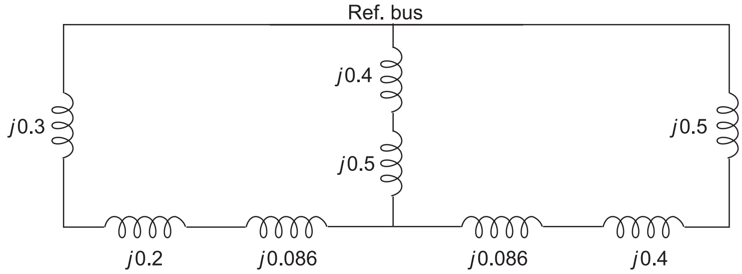
Line (each) $= 25 \times 50/(120)^2 = 0.086$,

$X_{LO} = 0.086 \times 2.5 = 0.215$

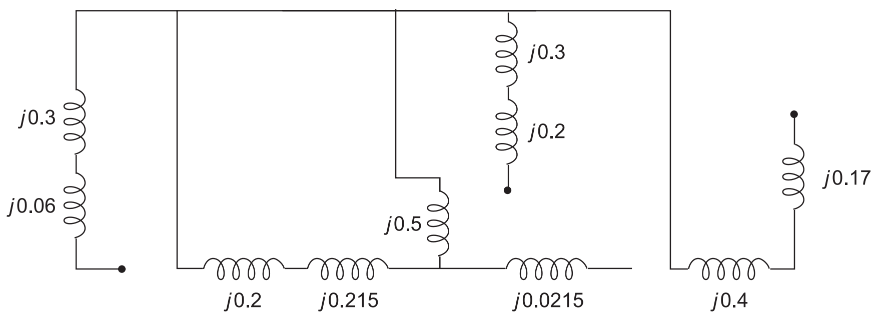
Neutral grounding reactance of G_1 and M_3 = 0.1 each



(a) Positive Sequence Network



(b) Negative Sequence Network



(c) Zero Sequence Network

Fig. S-10.9

Chapter 11

11.1 $I_{a1} = I_{a2} = I_{a0} = \dfrac{1}{j(0.2 + 0.3 + 0.1)} = -j\ 1.667$ pu

$$\text{Base current} = \frac{25}{\sqrt{3} \times 11} = 1.312 \text{ kA}$$

$\therefore$ $I_{a1} = -j\ 2.187$ kA; $I_a = -j$ **6.56** kA

$$V_{a1} = 1 - j\ 0.2 \times (-j\ 1.667) = 0.667$$

$$V_{a2} = -j\ 0.3 \times (-j\ 1.667) = -0.5$$

$$V_{a0} = -j\ 0.1 \times (-j\ 1.667) = -0.1667$$

$$V_a = 0$$

$$V_b = \alpha^2 \cdot V_{a1} + \alpha V_{a2} + V_{a0}$$

$$= 0.667 \angle -120° - 0.5 \angle 120° - 0.1667$$

$$= -0.25 - j\ 1.01$$

$$V_c = 0.667 \angle 120° - 0.5 \angle -120° - 0.1607$$

$$= -0.25 + j\ 1.01$$

$V_{bc} = V_b - V_c = -j\ 2.02$ pu $\qquad |V_{bc}| = 2.02 \times \dfrac{11}{\sqrt{3}} =$ **12.83** kV

$V_{ab} = V_a - V_b = 0.25 + j\ 1.01$ pu $\qquad |V_{ab}| = 1.04 \times \dfrac{11}{\sqrt{3}} =$ **6.61** kV

$V_{ca} = V_c - V_a = -0.25 + j\ 0.101$ pu $\qquad |V_{ca}| = 1.04 \times \dfrac{11}{\sqrt{3}} =$ **6.61** k/V

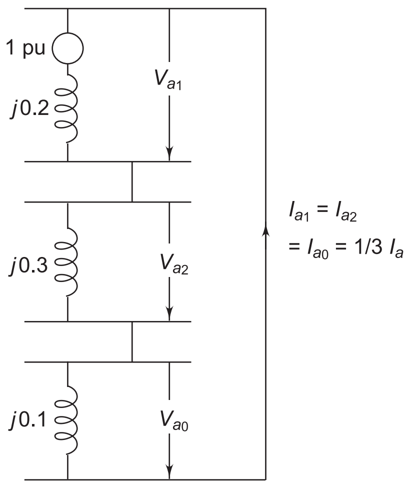


Fig. S-11.1

11.2 (a) *LL fault*

$$I_{a0} = 0;\ I_{a1} = -I_{a2} = \frac{1}{j0.5} = -j\mathbf{2}$$

$$I_b = -I_c = \alpha^2 I_{a1} + \alpha I_{a2}$$

$$= (\alpha^2 - \alpha)(-j2) = -2\sqrt{3}\ \text{pu}$$

$$V_{a1} = 1 - j\,0.2\,(-j2) = 0.6 = V_{a2}$$

$$V_{ab} = V_a - V_b$$

$$= (V_{a1} + V_{a2} + V_{a0}) - (\alpha^2 V_{a1} + \alpha V_{a2} + V_{a0})$$

$$= (2 - \alpha - \alpha^2) \times 0.6$$

$$= 1.8\ \text{pu} = 1.8 \times 11/\sqrt{3} = \mathbf{11.43}\ \text{kV} = V_{ac}$$

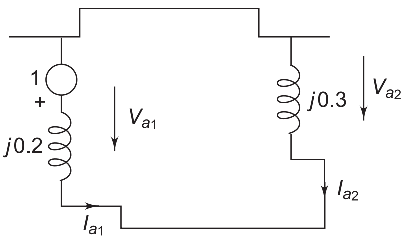


Fig. S-11.2 (a)

(b) *LLG fault*

$$I_{a1} = \frac{1}{j0.2 + (j0.3 \,||\, j0.1)} = -j\,3.64$$

$$I_{a2} = j\,3.64 \times 0.1/0.4 = j\,0.91$$

$$I_{a0} = j\,3.64 \times 0.3/0.4 = j\,2.73$$

$$V_{a1} = V_{a2} = V_{a0}$$

$$= 1 - (j\,0.2)(-j\,3.64) = 0.272$$

$$V_a = 3\,V_{a1} = 0.816;$$

$$V_b = 0;\ V_{ab} = V_a - V_b = \mathbf{0.816} = V_{ac}$$

$$I_b = \alpha^2 I_{a1} + \alpha I_{a2} + I_{a0}$$

$$= 3.64\ \angle 150^\circ + 0.91\ \angle -150^\circ + j\,2.73$$

$$= -3.94 + j\,4.1$$

$$|I_b| = \mathbf{5.69}\ \text{pu}$$

$$I_c = 3.64\ \angle 30^\circ + 0.91\ \angle -30^\circ + j\,2.73$$

$$= 3.94 + j\,4.1$$

$$\therefore\quad |I_c| = \mathbf{5.69}\ \text{pu}$$

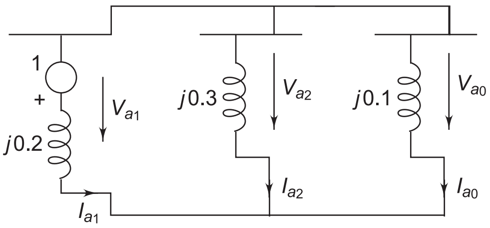


Fig. S-11.2 (b)

11.3 (i) LG fault $I_f = I_a = \dfrac{3}{j(0.2 + 0.2 + 0.08)} = -\,j\ 6.25$ pu

(ii) LL fault $I_f = I_b = -\,I_c = \dfrac{-j\sqrt{3} \times 1}{j0.4} = -\,4.33$ pu

(iii) LLG fault (Ref. Fig.S-11.2 b)

$$I_{a1} = \frac{1}{j0.2 + (j0.211\ j0.08)} = -\,j\ 3.89$$

$$I_{a2} = j\ 3.89 \times 0.08/0.28 = j\ 1.11$$

$$I_{a0} = j\ 3.89 \times 0.2/0.28 = j\ 2.78$$

$$I_b = 3.89\ \angle 150° + 1.11\ \angle -150° + j\ 2.78$$

$$= -\,4.33 + j\ 4.17$$

$\therefore$ $|I_b| =$ **6.01** pu,

$$I_c = 3.89\ \angle 30° + 1.11\ \angle -30° + j\ 2.78$$

$$= 4.33 + j\ 4.17$$

(iv) 3 phase fault $I_f = 1/j\ 0.2 = -\,j\ 5$ pu

In order of decreasing magnitude of line currents the faults can be listed:

(a) LG (b) LLG (c) 3 phase (d) LL

11.4 Let the neutral grounding resistance be X_n.

$$I_a = \frac{3}{j(0.2 + 0.2 + 0.08 + 3X_n)} = -\,j\ 5$$

$\therefore$ $X_n = 0.04$ pu

Base $Z = 121/25 = 4.84\ \Omega$

$\therefore$ $X_n = 4.84 \times 0.04 =$ **0.1936 Ω**

If grounding resistance is used (R_n)

$$|I_a| = \left|\frac{3}{j0.48 + 3R_n}\right| = 5 \text{ or } \frac{9}{9R_n^2 + 0.23} = 25$$

$\therefore$ $R_n = 0.12$ pu $= 0.12 \times 4.84 =$ **0.581 Ω**

With $X_n = 0.04$ included in generator neutral to ground:

$$Z_0 = j\ 0.08 + j\ 0.12 = j\ 0.2$$

LL fault

$$I_f = I_b = -\ I_c = \frac{-j\sqrt{3}\times 1}{j\,0.4} = -\ \mathbf{4.33}\ \text{pu}$$

$$\text{LLG fault } I_{a1} = \frac{1}{j\,0.2 + (j\,0.2\ ||\ j\,0.2)} = -\ j\ 3.334$$

$$I_{a2} = +\ j\ 1.667 = I_{a0}$$

$$I_b = 3.334\ \angle 150° + 1.667\ \angle -150° + j\ 1.667$$

$$= -\ 4.33 + j\ 2.5$$

$$|I_b| = 5\ \text{pu} \qquad I_f = 3I_{a0} = \boldsymbol{j\ 5}\ \text{pu}$$

11.5

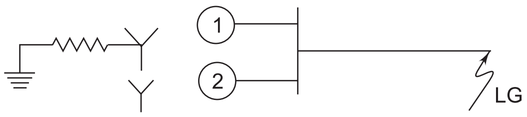


Fig. S-11.5 (a)

Base 25 MVA, 11 kV

Feeder reactances: Pos. sequence $\dfrac{j\,0.4\times 25}{121} = j\ 0.083$ pu

Neg. sequence $= j\ 0.083$ pu

Zero sequence $= j\ 0.166$ pu

Grounding resistance $= \dfrac{1\times 25}{121} = 0.207$ pu, $3R_n = 0.621$

Positive sequence network

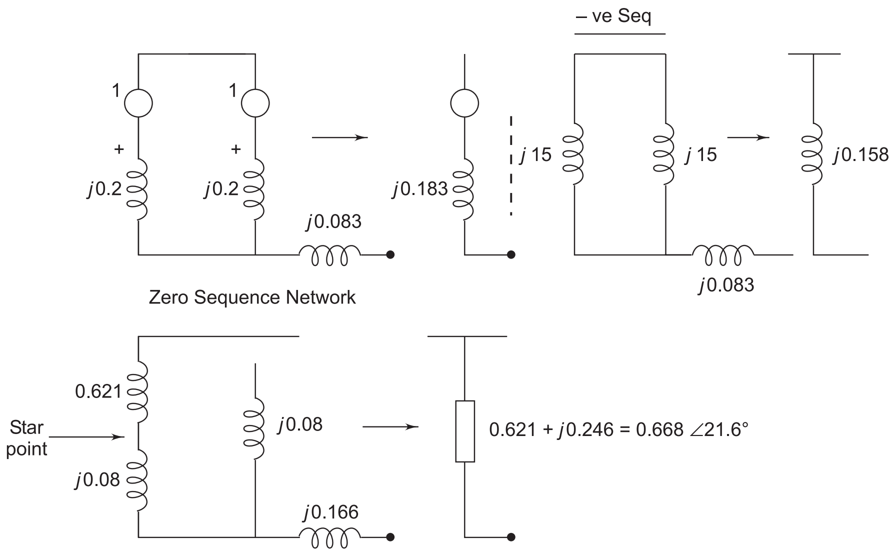


Fig. S-11.5 (b)

LG fault at feeder end

(a) $I^f = I_a = \dfrac{3}{0.621 + j\,0.587}$ or $|I^f| =$ **3.51** pu

(b) $I_{a1} = I_{a2} = I_{a0} = \dfrac{1}{0.621 + j\,0.587} = 1.17 \angle -43.4°$

$$V_{a1} = 1 - j\,0.183 \times 1.17 \angle -43.4° = 0.872 \angle -10.3°$$
$$V_{a2} = -j\,0.158 \times 1.17 \angle -43.4° = -0.184 \angle 46.6°$$
$$V_{a0} = -0.668 \angle 21.6° \times 1.17 \angle -43.4° = -0.782 \angle -21.8°$$
$$V_b = 0.872 \angle -130.3° - 0.184 \angle 166.6° - 0.782 \angle -21.8°$$
$$= \mathbf{1.19 \angle -159.5°}$$
$$V_c = 0.872 \angle 109.7° - 0.184 \angle -73.4° - 0.782 \angle -21.8°$$
$$= \mathbf{1.68 \angle 129.8°}$$

(e) Voltage of star point w.r.t. ground $= 3I_{a0} \times 0.207$

$= 3 \times 1.17 \times 0.207$

$=$ **0.726** pu

11.6 Since the star point is isolated from ground LLG fault is just like LL fault.

$$I_b = -I_c = \frac{-j\sqrt{3} \times 1}{j\,0.35 + j\,0.25} = \mathbf{-\,2.887} \text{ pu}$$

11.7

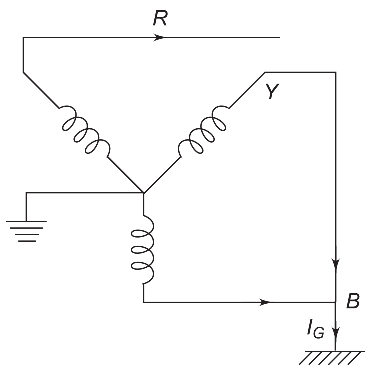


Fig. S-11. 7

$$V_{R1} = \frac{11}{\sqrt{3}} \angle 0° \text{ kV} = 6351 \text{ volts}$$

Neutral solidly grounded (See Fig. S-11.2 b)

$$I_{R1} = \frac{6{,}351}{j1 + (j0.8 \| j0.4)} = -j\;5{,}013 \text{ A}$$

$$I_{R2} = j\;5{,}013 \times \frac{0.4}{1.2} = j\;1{,}671$$

$$I_{R0} = j\,5{,}013 \times \frac{0.8}{1.2} = j\,3{,}342$$

$$I_Y = \alpha^2 I_{R1} + \alpha I_{R2} + I_{R0}$$
$$= 5013 \angle 150° + 1671 \angle -150° + j\,3{,}342 = -\mathbf{5.79} + \boldsymbol{j}\,\mathbf{5.01}\text{ kA}$$

$$I_B = \alpha I_{R1} + \alpha^2 I_{R2} + I_{R0} = 5013 \angle 30° + 1671 \angle -30° + j\,3{,}342$$
$$= \mathbf{5.79 + j\,5.01\ kA}$$

$$I_G = I_Y + I_B = j\,\mathbf{10.02\ kA};\ \boldsymbol{I_R} = \mathbf{0}$$

(b) This is equivalent to LL case

$$I_B = -I_Y = (-j\sqrt{3} \times 6{,}351)/j1.8 = -\mathbf{6.111}\text{ kA}$$
$$I_G = \mathbf{0}\text{ A.}$$

11.8 Base: 10 MVA, 3.3 kV (gen and line), 0.6 kV (motors)

Motor MVA = $\frac{5}{0.9}$ = 5.56 (Total). Let there be n motors.

$\therefore$ Rating of each motor = $\frac{5.56}{n}$ MVA, 0.6 kV;

$$X'' = X_2 = 20\%,\ X_0 = 5\%.$$

Rating of eqv. motor = 5.56 MVA, 0.6 kV, $X'' = X_2 = \frac{20}{n} \times \frac{5.56}{\frac{5.56}{n}} = 20\%$

Motor reactance to base of 10 MVA

$$X_0 = 5\%\ X_n = 2.5\%\text{ on eqv. motor rating}$$

$$X'' = X_2 = 0.2 \times \frac{10}{5.56} = 0.36\text{ pu;}$$

$$X_0 = 0.05 \times \frac{10}{5.56} = 0.09\text{ pu}$$

$$X_n = 0.025 \times \frac{10}{5.56} = 0.045$$

Motor load: 4/10 = 0.4 pu (MW): 1 pu voltage, 0.8 lag pf

Prefault motor current = $\frac{0.4}{0.9 \times 0.8 \times 1}$ = 0.556 $\angle -36.9°$ pu

Generator reactance $X'' = X_2 = 0.1$ pu, $X_0 = 0.05$ pu

Transformer reactance $X = 0.1 \times 10/7.5 = 0.133$ pu

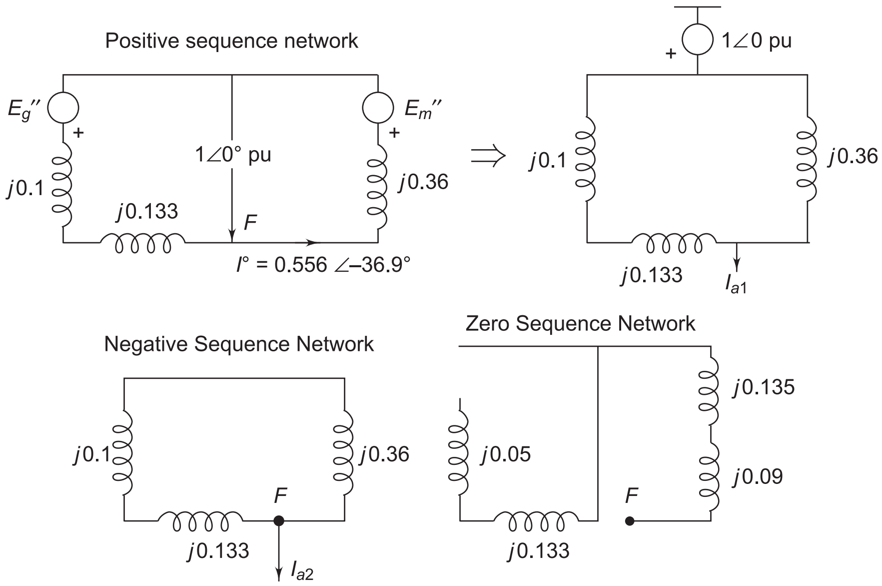


Fig. S-11.8(a) Connection of sequence networks to simulate the fault (LG)

$$E_g'' = 1 + j\,0.233 \times 0.556 \;\angle -36.9°$$
$$= 1 + 0.13 \;\angle 53.1° = 1.08 \;\angle 5.5°$$
$$E_m'' = 1 - j\,0.36 \times 0.556 \;\angle -36.9° = 0.89 \;\angle -10.3°$$

Connection of sequence networks to simulate the fault (LG)

It immediately follows from sequence network connection that

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{j(0.1414 + 0.1414 + 0.225)}$$
$$= -j\,1.97$$
$$I^f = 3 \times -j\,1.97 = -j\,5.91 \text{ pu}$$
$$I_{ag1} = -j\,1.97 \times \frac{0.36}{0.593} = -j\,1.20$$
$$I_{ag2} = -j\,1.2; \; I_{ag0} = 0$$

Positive sequence and negative sequence currents on star side are shifted by + 90° and – 90° respectively from delta side.

$$\therefore \qquad I_{ag1} = 1.20 \;\; I_{ag2} = -1.2, \;\; I_{ag0} = 0$$

$$I_{am1} = -j\,1.97 \times \frac{0.233}{0.593} = -j\,0.77$$

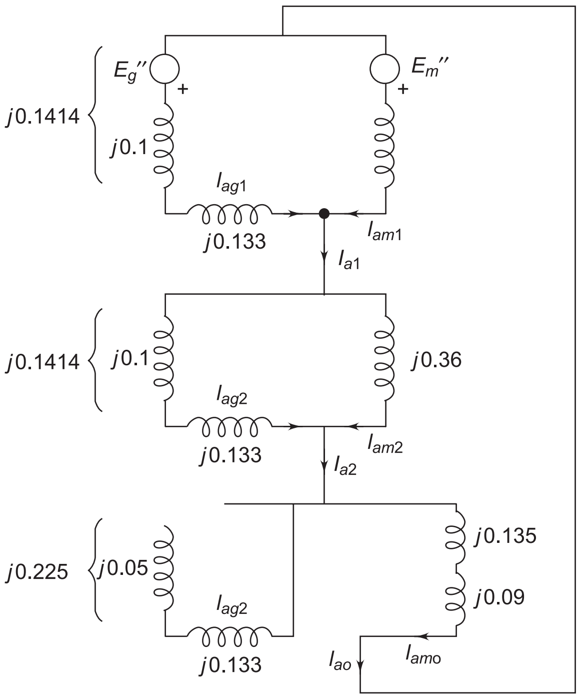


Fig. S-11.8 (b)

$I_{am2} = -j\ 0.77;\ I_{am0} = -j\ 1.97$

$I_{am} = -j\ \mathbf{3.51}$ pu

$I_{bm} = (\alpha^2 + \alpha)(-j\ 0.77) - j\ 1.97 = -j\ \mathbf{1.20}$ pu

$I_{cm} = (\alpha + \alpha^2)(-j\ 0.77) - j\ 1.97 = -j\ \mathbf{1.20}$ pu

$I_{ag} = \mathbf{0}$ pu

$I_{bg} = (\alpha^2 - \alpha) \times 1.2 = -j\ \mathbf{2.08}$ pu

$I_{cg} = (\alpha - \alpha^2) \times 1.2 = j\ \mathbf{2.08}$ pu

11.9 Equivalent seq. reactances are

$X_1 = j\ 0.105$ pu

$X_2 = j\ 0.105$ pu

$$X_0 = j\ \frac{0.05 \times 0.45}{0.5} = j\ 0.045 \text{ pu}$$

$$I_{a1} = \frac{1}{j\,0.105 + (j\,0.105\,||\,j\,0.045)}$$

$= -j\ 7.33$

$$I_{a2} = j\ 7.33 \times \frac{0.045}{0.15} = j\ 2.20$$

$$I_{a0} = j\ 5.13$$

$$I_{a1}^1 = -\ j\ 7.33 \times \frac{0.35}{0.5} = -\ j\ 5.131$$

$$I_{a2}^1 = j\ 2.2 \times \frac{0.35}{0.5} = j\ 1.54;\ I_{a0}^1 = 0$$

In the generator

$$I_{a1}^1 = j\ (-\ j\ 5.131) = 5.131;\ I_{a2}^1 = -\ j\ (j\ 1.54) = 1.54$$

$$I_{c1}^1 = \alpha\ I_{a1}^1 = -\ 2.566 + j\ 4.443;$$

$$I_{c2}^1 = \alpha^2\ I_{a2}^1 = -\ 0.77 - j\ 1.333$$

$$\therefore \quad I_c^1 = I_{c1}^1 + I_{c2}^1 = -\ 3.336 + j\ 3.11$$

$$\therefore \quad |I_c^1| = 4.56 \text{ pu; Base current } = \frac{1200 \times 1000}{\sqrt{3} \times 600} = 1{,}155\ A$$

$$\therefore \quad |I_c^1| = 4.56 \times 1{,}155 = \mathbf{5{,}266}\ A$$

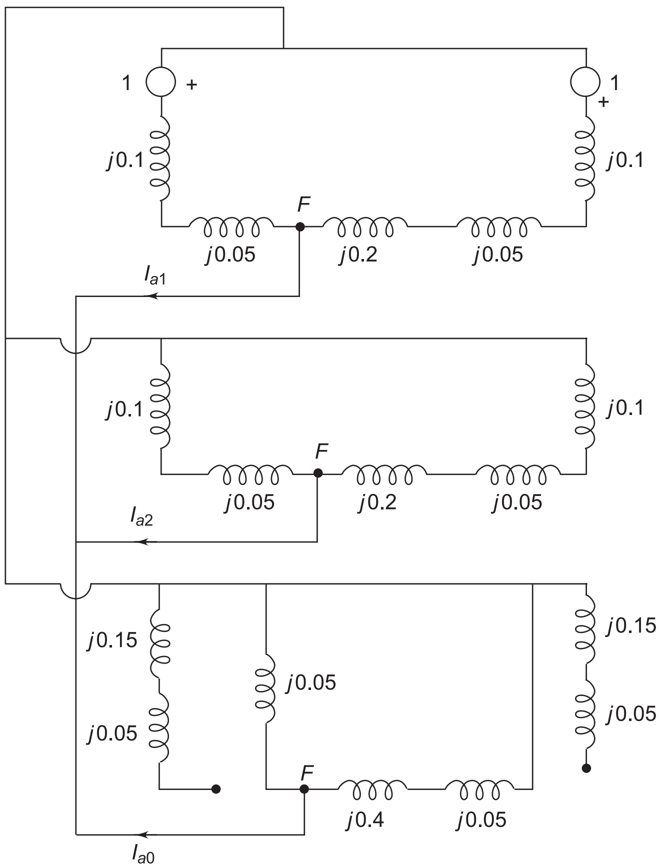


Fig. S-11.9

11.10

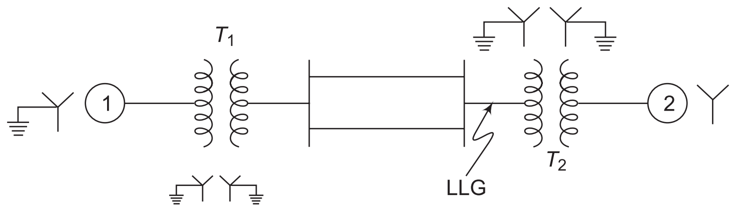


Fig. S-11.10 (a)

Equivalent Sequence reactances are

$$X_1 = \frac{0.65 \times 0.4}{1.05} = 0.248;$$

$$X_2 = \frac{0.55 \times 0.3}{0.85} = 0.194; \; X_0 = j\ 0.55$$

$$I_{a1} = \frac{1}{j\,0.248 + (j\,0.194 \,||\, j\,0.55)} = -\ j\ 2.55$$

$$I_{a2} = j\ 2.55 \times \frac{0.55}{0.744}$$

$$= j\ 1.885$$

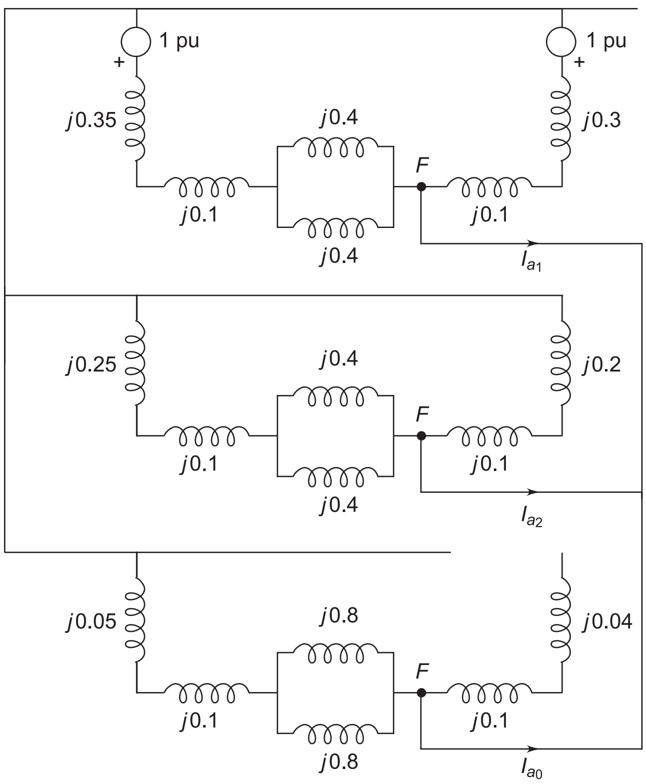


Fig. S-11.10 (b)

$$I_{a0} = j\ 0.665$$
$$I_b = 2.55\ \angle 150° + 1.885\ \angle -150° + j\ 0.665$$
$$= -3.84 + j\ 1.0$$
$$I_c = 2.55\ \angle 30° + 1.885\ \angle -30° + j\ 0.665$$
$$= 3.84 + j\ 1.0$$
$$I_f = I_b + I_c = j\ 2.0 \text{ pu}$$

11.11

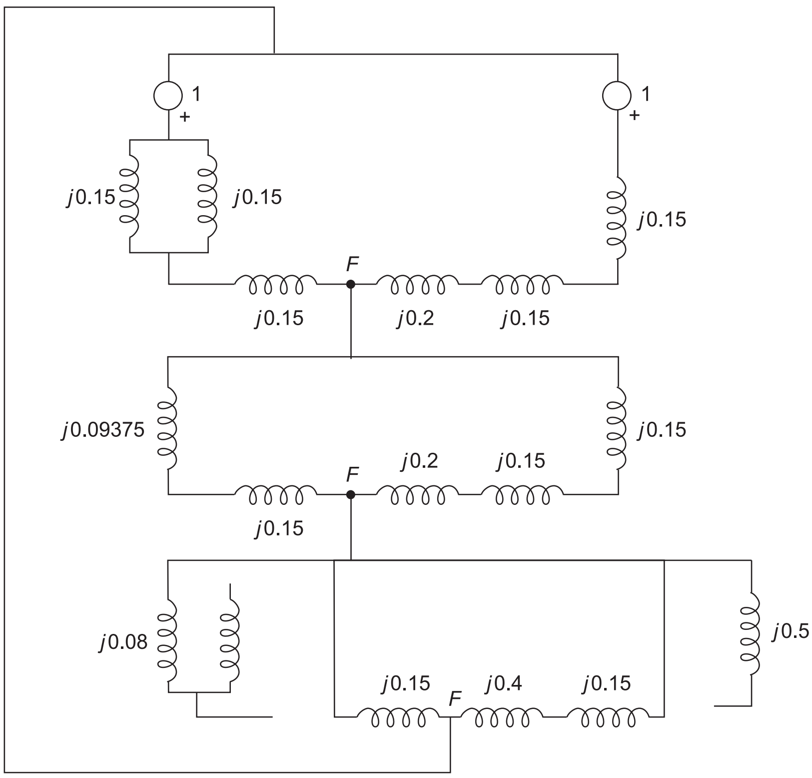


Fig. S-11.11

Equivalent sequence reactances are:

$$X_1 = 0.1638$$
$$X_2 = 0.1638$$
$$X_0 = 0.118$$
$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{j\,0.4456}$$
$$= -j\ 2.244$$
$$\therefore \quad I^f = 3I_{a1} = -j\ \mathbf{6.732}$$

Sequence currents through transformer *A*

$$I_{a1}\ (A) = I_{a2}\ (A) = -\ j\ 2.244 \times \frac{0.5}{0.744} = -\ j\ 1.508$$

$$I_{a0}\ (A) = -\ j\ 2.244 \times \frac{0.55}{0.7} = -\ j\ 1.763$$

$$I_a\ (A) = -\ j\ 1.508 - j\ 1.508 - j\ 1.763 = -\ j\ \mathbf{4.779}\ \text{pu}$$

$$I_b\ (A) = 1.508\ \angle 150° + 1.508\ \angle 30° - j\ 1.763 = -\ j\ \mathbf{0.225}\ \text{pu}$$

$$I_c\ (A) = 1.508\ \angle 30° + 1.508\ \angle 150° - j\ 1.763 = -\ j\ \mathbf{0.255}\ \text{pu}$$

11.12.

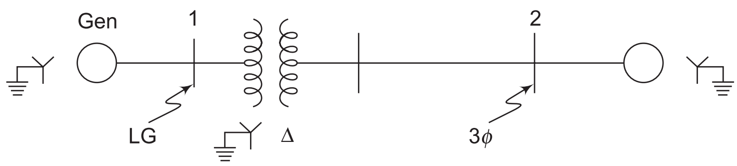


Fig. S.11.12 a

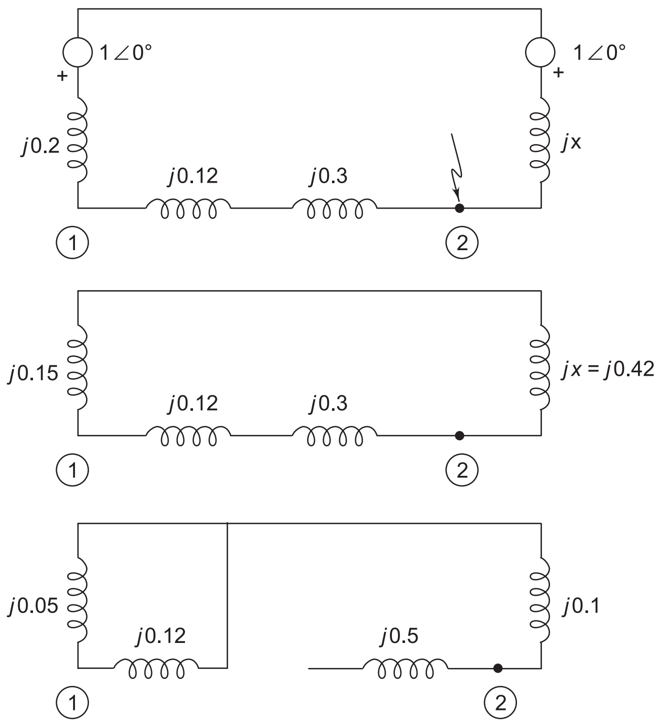


Fig. S.11.12 b

3 phase short at bus 2 (F):

$$\frac{1}{0.62} + \frac{1}{X} = 4$$

$$X = \mathbf{0.42}\ \text{pu}$$

LG fault at bus 1:

Equivalent sequence reactance are:

$$X_1 = \frac{0.2 \times 0.84}{1.04} = 0.1615$$

$$X_2 = \frac{0.15 \times 0.84}{0.99} = 0.1273$$

$$X_0 = \frac{0.05 \times 0.12}{0.17} = 0.0353$$

$$I_f = 3\ I_{a1} = \frac{3 \times 1}{j\,0.3241} = -\ j\ \mathbf{9.256}\ \text{pu}$$

11.13

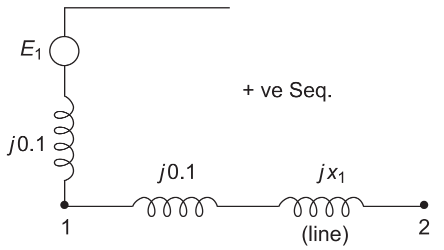


Fig. S-11.13 (a)

$$Z_{\text{BUS1}} = Z_{\text{BUS2}} = \begin{bmatrix} j\,0.1 & j\,0.1 \\ j\,0.1 & j\,(0.2 + X_1) \end{bmatrix}$$

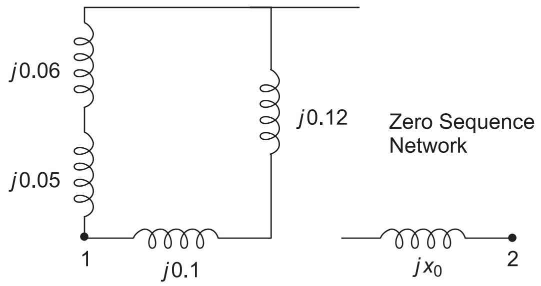


Fig. S-11.13 (b)

$$Z_{\text{BUS0}} = \begin{bmatrix} j\,0.069 & 0 \\ 0 & \infty \end{bmatrix}$$

The fault current with LG fault on bus 1 is

$$I_1^f = \frac{3 \times 1}{j\,0.1 + j\,0.1 + j\,0.069} = -\ j\ \mathbf{11.152}\ \text{pu}$$

From Fig. S-11.13 c, it is clear that all I_{a1} and I_{a2} flow towards bus 1 from the generator only. The component of I_{a0} flowing towards bus 1 from generator is

$$\left(\frac{-j\,11.152}{3}\right)\times\frac{j\,0.22}{j\,0.11+j\,0.22}$$

$$= -\,j\ 3.717\ \times 2/3 = -\ j\ \mathbf{2.478}\ \text{pu}$$

and the component of I_{a0} flowing towards bus 1 from transformer is

$$-\,j\ 3.717\ \times\ \frac{j\,0.11}{j\,0.11+j\,0.22}\ = -\,j\ \mathbf{1.239}\ \text{pu}$$

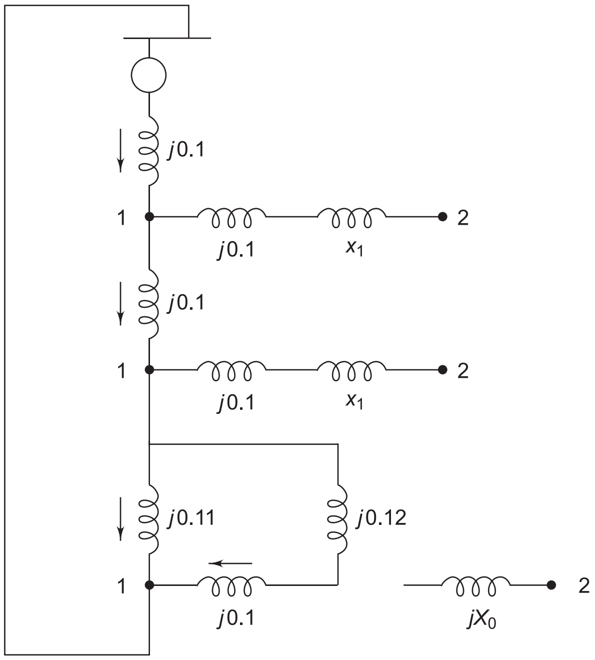


Fig. S-11.13 (c)

11.14 Equivalent Seq. reactances are:

$$X_1 = \frac{0.4445\times 0.3105}{0.755} = j\ 0.1828$$

$$X_2 = j\ 0.1828$$

$$I_{a1} = -\,I_{a2} = \frac{1}{j\,0.3656} - j\ 2.735$$

$$I_b = -\,I_c = \frac{-j\sqrt{3}\times 1}{j\,0.3656} = -\ 4.737\ \text{pu}$$

$$|I_f| = \mathbf{4.737}\ \text{pu}$$

$$V_{a2} = -\,I_{a2}\ z_2 = -\ j\ 2.735\ \times j\ 0.1828$$

$$= 0.5\ \text{pu}$$

$\therefore$ $V_{a1} = 0.5$ pu and $V_{a0} = 0$ $(\because\ I_{a0} = 0)$

∴ Voltage of healthy phase

$$V_a = V_{a1} + V_{a2} + V_{a0} = \mathbf{1} \text{ pu}$$

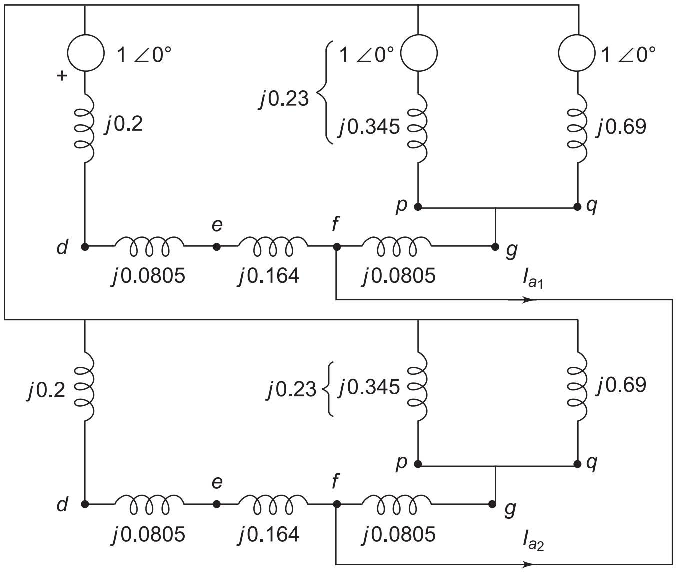


Fig. S-11.14

11.15 From Example 9.6, we have

$$Z_{1\text{BUS}} = Z_{2\text{BUS}} = j\begin{bmatrix} 0.0903 & 0.0597 & 0.0719 & 0.0780 \\ 0.0597 & 0.0903 & 0.0780 & 0.0719 \\ 0.0719 & 0.0780 & 0.1356 & 0.0743 \\ 0.0780 & 0.0719 & 0.0743 & 0.1356 \end{bmatrix}$$

From the data given, zero sequence reactance diagram is drawn below.

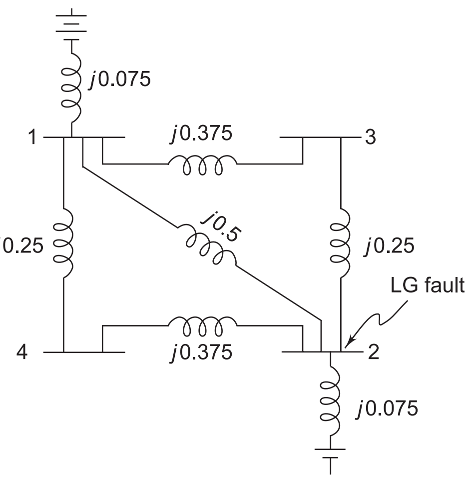


Fig. S-11.15

$$Y_{0BUS} = \begin{bmatrix} -j22 & j2 & j2.667 & j4 \\ j2 & -j22 & j4 & j2.667 \\ j2.667 & j4 & -j6.667 & j0 \\ j4 & j2.667 & j0 & -j6.667 \end{bmatrix}$$

$$Z_{0BUS} = j\begin{bmatrix} 0.0585 & 0.0164 & 0.0332 & 0.0417 \\ 0.0164 & 0.0585 & 0.0417 & 0.0332 \\ 0.0332 & 0.0417 & 0.1883 & 0.0366 \\ 0.0417 & 0.0332 & 0.0366 & 0.1883 \end{bmatrix}$$

From Eq. (11.47)

$$I_2^f = \frac{3 \times 1}{j0.0903 + j0.0903 + j0.0585} = -j\,\mathbf{12.547}\text{ pu}$$

$$I_{1-2}^f = I_{2-2}^f = I_{0-2}^f = -j\,4.182$$

From Eq. (11.49)

$$V_{1-1}^f = V_{1-1}^0 - Z_{1-12}\,I_{1-2}^f$$
$$= 1 - j\,0.0597 \times -j\,4.182 = 0.7503$$
$$V_{1-2}^f = V_{1-2}^0 - Z_{1-22}\,I_{1-2}^f$$
$$= 1 - j\,0.0903 \times -j\,4.182 = 0.6224$$
$$V_{2-1}^f = V_{2-1}^0 - Z_{2-12}\,I_{2-2}^f$$
$$= 0 - j\,0.0597 \times -j\,4.182 = -0.2497$$
$$V_{2\text{-}2}^f = V_{2-2}^0 - Z_{2-22}\,I_{2-2}^f$$
$$= 0 - j\,0.0903 \times -j\,4.182 = -0.3776$$
$$V_{0-1}^f = V_{0-1}^0 - Z_{0-12}\,I_{0-2}^f$$
$$= 0 - j\,0.0164 \times -j\,4.182 = -0.0686$$
$$V_{0-2}^f = V_{0-2}^0 - Z_{0-22}\,I_{0-2}^f$$
$$= 0 - j\,0.0585 \times -j\,4.182 = -0.2446$$
$$V_1^f\,(a) = 0.7503 - 0.2497 - 0.0686 = 0.432$$
$$V_2^f\,(a) = 0.6224 - 0.3776 - 0.2446 = 0$$

(LG fault is on bus 2 phase a)

$$I_{12}^f\,(a) = 0.864$$
$$V_1^f\,(b) = 0.7503\ \angle -120° - 0.2497\ \angle 120° - 0.0686$$
$$= -0.3189 - j\,0.866$$
$$V_2^f\,(b) = 0.6224\ \angle -120° - 0.3776\ \angle 102° - 0.2446$$
$$= -0.367 - j\,0.866$$
$$I_{12}^f\,(b) = \frac{V_1^f(b) - V_2^f(b)}{j0.5}$$
$$= \frac{0.0481}{j0.5} = -j\,\mathbf{0.0962}\text{ pu}$$

Similarly other voltages and currents can be calculated.

Chapter 12

12.1 Moment of inertia of rotor = 10,000 kg–m^2

Rotor speed = 3,000 rpm = $\frac{3{,}000 \times 2\pi}{60}$ = 100 π rad/sec

$$GH = \frac{1}{2} I\omega^2;\ \frac{100}{0.85} \times H$$

$$= \frac{1}{2} \times 10^4 \times 10^4 \times \pi^2 \times 10^{-6}$$

$$\therefore \quad H = \frac{100 \times \pi^2 \times 0.85}{100 \times 2} = \mathbf{4.19}\ \text{MJ/MVA}$$

$$M = GH/180f = \frac{4.19 \times 100}{180 \times 50 \times 0.85} = \mathbf{0.0547}\ \text{MJ-sec/elec. deg}$$

12.2 *m*/*c* 1 : ω = 1,500 rpm = 50 π rad/sec

$$\frac{60}{0.8} \times H_1 = \frac{1}{2} \times 3 \times 10^4 \times 2{,}500 \times \pi^2 \times 10^{-6}$$

$$\therefore \quad H_1 = 4.93\ \text{MJ/MVA}$$

m/*c* 2: ω = 3,000 rpm = 100 π rad/sec.

$$\frac{80}{0.85} \times H_2 = \frac{1}{2} \times 10^4 \times 10^4 \times \pi^2 \times 10^{-6}$$

$$\therefore \quad H_2 = 5.24\ \text{MJ/MVA}$$

$$\therefore \quad H_{eq} = \frac{4.93 \times 60}{0.8 \times 200} + \frac{5.24 \times 80}{0.85 \times 200} = \mathbf{4.315}\ \text{MJ/MVA}$$

(Base: 200 MVA)

12.3 Heq = $4 \times \frac{7 \times 80}{100} + 3 \times \frac{3 \times 200}{100}$ = **40.4** MJ/MVA (Base: 100 MVA)

12.4 $R = 0.11 \times 500 = 55\ \Omega$; $X = 1.45 \times 10^{-3} \times 314 \times 500 = 227.7\ \Omega$

$$Z = 55 + j\,227.7 = 234.2\ \angle 76.4°;$$

$$Y = 314 \times 0.009 \times 10^{-6} \times 500\ \angle 90° = 0.0014\ \angle 90°$$

$$A = 1 + \frac{1}{2} YZ = 1 + \frac{1}{2} \times 0.0014 \times 234.2\ \angle 166.4°$$

$$= 0.841\ \angle 2.6°$$

$$B = Z\left(1 + \frac{1}{6} YZ\right)$$

$$= 234.2\ \angle 76.4° + \frac{1}{6} \times 0.0014 \times (234.2)^2\ \angle -117.2°$$

$$= 221.7\ \angle 77.2°$$

$$P_{e,max} = \frac{|E||V|}{|B|} - \frac{|A||V|^2}{|B|} \cos(\beta - \alpha)$$

$$= \frac{(200)^2}{221.7} - \frac{0.841 \times (200)^2}{221.7} \cos 74.6°$$

$$= \mathbf{140.1} \text{ MW}$$

Capacitance neglected

$$A = 1\angle 0, \; B = 234.2 \; \angle 76.4°;$$

$$P_{e,\,max} = \frac{(200)^2}{234.2}(1 - \cos 76.4°)$$

$$= \mathbf{130.63} \text{ MW}$$

Capacitance and resistance neglected

$$A = 1\angle 0° \qquad B = 227.7 \; \angle 90°$$

$$P_{e,\,max} = \frac{(200)^2}{227.7}(1 - \cos 90°] = \mathbf{175.67} \text{ MW}$$

12.5

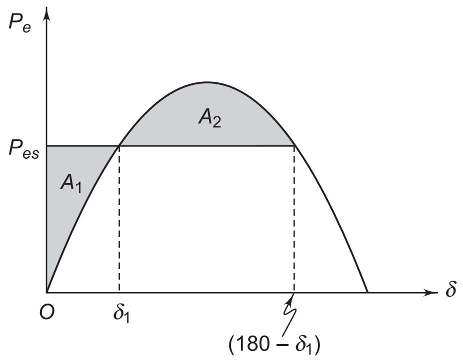


Fig. S-12.5

$$P_e = 100 \sin \delta$$

Max. load that can be suddenly switched on = $P_{es} = 100 \sin \delta_1$

By equal area criterion

$$\int_0^{\delta_1} (P_{es} - 100 \sin \delta)\, d\delta = \int_{\delta_1}^{\pi - \delta_1} (100 \sin \delta - P_{es})\, d\delta$$

$$P_{es}\, \delta_1 + 100 \cos \delta\Big|_0^{\delta_1} = -100 \cos \delta\Big|_{\delta_1}^{\pi-\delta_1} - P_{es}\, \delta\Big|_{\delta_1}^{\pi-\delta_1}$$

$$100\, \delta_1 \sin \delta_1 + 100 \cos \delta_1 - 100 = 200 \cos \delta_1 - 314 \sin \delta_1$$
$$+ 200\, \delta_1 \sin \delta_1$$

$$\frac{100 \times \pi}{180}\, \delta_1 \sin \delta_1 + 100 \cos \delta_1 - 314 \sin \delta_1 + 100 = 0$$

It is a nonlinear eqn. which has to be solved numerically or graphically.

$$1.745\ \delta_1 \sin \delta_1 + 100 \cos \delta_1 - 314 \sin \delta_1 + 100 = 0$$

$\therefore \quad \delta_1 = 46.5° \ \therefore\ P_{es} = 100 \sin 46.5° = \mathbf{72.54}$ MW

12.6

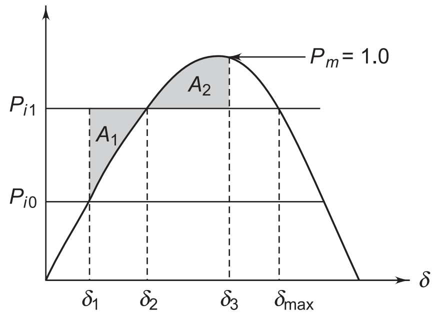


Fig. S-12.6

$$P_{i0} = 0.3 = \sin \delta_1$$

$$\therefore \quad \delta_1 = 17.5°$$

$$P_{i1} = 0.6;\ \delta_2 = 36.9°$$

$$A_1 = \left[0.6\,(\delta_2 - \delta_1) - \int_{\delta_1}^{\delta_2} \sin\delta\, d\delta\right] = 0.049$$

$$A_2 = z\left[\int_{\delta_2}^{\delta_3} \sin\delta\, d\delta - 0.6\,(\delta_3 - \delta_2)\right]$$

$$A_2 - A_1 = \left[\int_{\delta_1}^{\delta_3} \sin\delta\, d\delta - 0.6\,\delta_3 + 0.6\,\delta_1\right]$$

$$\therefore \int_{\delta_1}^{\delta_3} \sin\ \delta\, d\delta - 0.6\ (\delta_3 - \delta_1) = 0$$

$$\therefore \quad \cos \delta_3 + 0.6\ \delta_3 = \cos 17.5° + \frac{0.6 \times 17.5 \times \pi}{180}$$

$$\cos \delta_3 + 0.6\ \delta_3 = 0.954 + 0.183 = 1.137$$

By trial and error procedure, we find $\delta_3 = \mathbf{58°}$

Synchronism will not be lost.

$$\delta_{max} = 180° - \delta_2$$
$$= 180 - 36.9$$
$$= 143.1°$$

$$A_{2,\ max} = \int_{\delta_2}^{\delta_{max}} \sin\ \delta\, d\delta - 0.6\ (\delta_{max} - \delta_2)$$

$$= -\cos\ \delta\Big|_{\delta_2}^{\delta_{max}} - 0.6\ (\delta_{max} - \delta_2)$$

$$= -\cos 143.1° + \cos 36.9° - 0.6\,(143.1 - 36.9) \times \frac{\pi}{180}$$

$$= 0.487$$

$\therefore\ A_{2,\,\max} > A_1$

$\therefore$ System is stable

Excursion about the new steady state rotor position

$$= \delta_3 - \delta_2 = 58 - 36.9 = \mathbf{21.1°}$$

12.7 P_{eI} (prefault) $= \dfrac{(200)^2}{150} \sin \delta$

$$= 266.7 \sin \delta$$

P_{eII} (during fault) $= \dfrac{(200)^2}{400} \sin \delta$

$$= 100 \sin \delta$$

P_{eIII} (post fault) $= \dfrac{(200)^2}{200} \sin \delta$

$$= 200 \sin \delta$$

Max. load transfer corresponds to $A_1 = A_2$

$$A_1 = \int_{\delta_1}^{\delta_1 + 60°} (P_i - 100 \sin \delta)\, d\delta = P_i \times \frac{\pi}{180} \times 60° + 100\,[\cos (\delta_1 + 60°) - \cos \delta_1]$$

Now $\quad P_i = 266.7 \sin \delta_1$

$\therefore \quad A_1 = (\pi/3) \times 266.7 \sin \delta_1 + 100 \cos (\delta_1 + 60°) - 100 \cos \delta_1$

$\quad = 279.3 \sin \delta_1 + 100 \cos (\delta_1 + 60°) - 100 \cos \delta_1$

Now $\quad \delta_2 = 180° - \sin^{-1} (P_i/200) = 180° - \sin^{-1} \left(\dfrac{266.7}{200} \sin \delta_1 \right)$

$$A_2 = \int_{\delta_1 + 60}^{\delta_2} (200 \sin \delta - P_i)\, d\delta$$

$$= -200 \cos \delta \Big|_{\delta_1 + 60}^{\delta_2} - P_i\,(\delta_2 - \delta_1 - 60°) \times \pi/180$$

$$= -200 \cos \delta_2 + 200 \cos (\delta_1 + 60°)$$

$$= 4.65\,(\delta_2 - \delta_1 - 60°) \sin \delta_1$$

$A_1 = A_2$

$279.3 \sin \delta_1 + 100 \cos (\delta_1 + 60°) - 100 \cos \delta_1 = -200 \cos \delta_2 +$
$\quad 200 \cos (\delta_1 + 60°) - 4.65\,(\delta_2 - \delta_1 - 60°) \sin \delta_1$

where $\quad \delta_2 = 180° - \sin^{-1} (1.334 \sin \delta_1)$

$$279.3 \sin \delta_1 - 100 \cos (\delta_1 + 60°) - 100 \cos \delta_1 + 200 \cos \delta_2 + 4.65$$
$$(\delta_2 - \delta_1 - 60°) \times \sin \delta_1 = 0$$

Solving $\delta_1 = 28.5°$

$\therefore$ $P_{i(\max)} = 266.7 \sin 28.5° =$ **127.3** MW

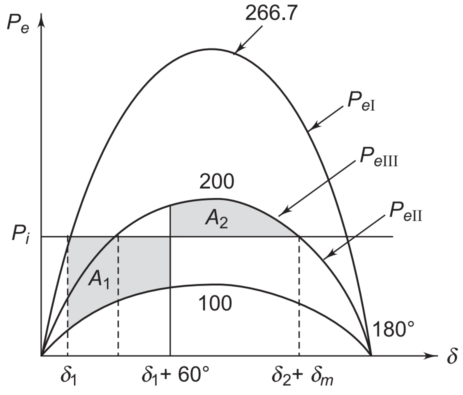


Fig. S-12.7

12.8 $\delta_1 = \sin^{-1} \dfrac{250}{500} = 30°$

$$\delta_2 = \sin^{-1} \frac{250}{350} = 45.6°$$

$\therefore$ $\delta_m = 180° - 45.6° = 134.4°$

$$A_1 = \frac{\pi}{180} \times (\delta_c - 30°) \times 250$$
$$= 4.36 \ \delta_c - 130.9$$

$$A_2 = \int_{\delta_c}^{\delta_m} (350 \sin \delta - 250) \, d\delta$$
$$= 350 \cos \delta_c + 4.36 \ \delta_c - 341.5$$

For δ_c

$$4.36 \ \delta_c - 130.9 = 350 \cos \delta_c + 4.36 \ \delta_c - 341.5$$
$$\cos \delta_c = 210.6/350 \ \therefore \ \delta_c = \mathbf{53°}$$

Swing eqn. upto critical clearing angle is

$$\frac{d^2\delta}{dt^2} = 250/M \text{ or } \frac{d\delta}{dt} = \frac{250}{M} t$$

$\therefore$ $$\delta = \frac{1}{2} \frac{250}{M} t^2 + \delta_1$$

$$\delta_c - \delta_1 = (\pi/180)\ 23° = \frac{125}{M}\ t_c^2$$

$$t_c^2 \text{ (critical clearing time)} = \frac{23 \times \pi}{180 \times 125}\ M$$

$$\therefore \qquad t_c = \sqrt{0.0032\ M} = 0.056\ \sqrt{M} \text{ sec}$$

We need to know the *inertia constant M to determine* t_c.

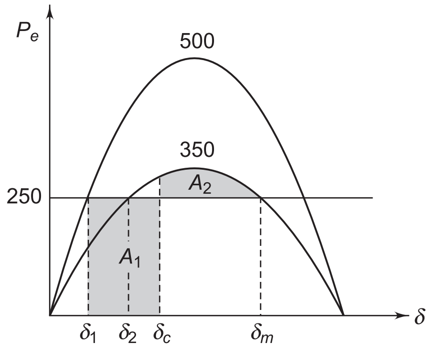


Fig. S-12.8

12.9

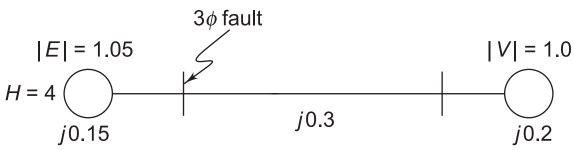


Fig. S-12.9

I. Prefault $X_{\text{I}} = 0.15 + 0.3 + 0.2 = 0.65$

$$P_{eI} = \frac{1 \times 1.05}{0.65} \sin \delta = 1.615 \sin \delta; \text{ Prefault power} = 1$$

$$\delta_0 = \sin^{-1} \frac{1}{1.615} = 38.3°$$

II. During fault $P_{e\text{II}} = 0$

III. Post fault $P_{e\text{III}} = P_{e\text{I}} = 1.615 \sin \delta$

Time to clear the fault = 0.1 sec; $M = \frac{GH}{180\ f} = \frac{1 \times 4}{180 \times 50} = 4.44 \times 10^{-4}$ sec^2/elec deg

Let us choose $\Delta t = 0.025$ sec.

$$\frac{(\Delta t)^2}{M} = \frac{(0.025)^2}{4.44 \times 10^{-4}} = 1.41;$$

$$\delta_m = \pi - \sin^{-1} (1/1.615) = 141.7°$$

t *sec*	P_m	$P_e = P_m \sin \delta$	$P_a = 1 - P_e$	$1.41\ P_a$	$\Delta\delta$ *deg*	δ *deg*
0–	1.615	1.0	0	0	–	38.3
0+	0	0	1.0	1.41	–	38.3
0av	–	–	0.5	0.705	0.705	38.3
0.025	0	0	1.0	1.41	2.115	39.005
0.05	0	0	1.0	1.41	3.525	41.12
0.075	0	0	1.0	1.41	4.935	44.645
1.0–	0	0	1.0	–	–	49.58
1.0+	1.615	1.23	– 0.23			49.58
1.0_{av}			0.385	0.543	5.478	49.58
1.025	1.615	1.324	– 0.324	– 0.456	5.022	55.058
1.05	1.615	1.4	– 0.4	– 0.564	4.458	60.08
1.075	1.615	1.46	– 0.46	– 0.649	3.809	64.538
2.0	1.615	1.501	– 0.501	– 0.706	3.103	68.347
2.025	1.615	1.531	– 0.531	– 0.750	2.353	71.45
2.05	1.615	1.551	– 0.551	– 0.777	1.576	73.803
2.075	1.615	1.563	– 0.563	– 0.794	0.782	75.379
3.0	1.615	1.568	– 0.568	– 0.8	– 0.018	76.161
3.025	1.615	1.568	– 0.568	– 0.8	– 0.818	76.143
3.05	1.615	1.562	– 0.562	– 0.792	– 1.61	75.325
3.075	1.615	1.55	– 0.55	– 0.776	– 2.386	73.71

After fault clearance δ goes through a maximum and then begins to reduce, the system is *stable*.

12.10 From Eq. (12.67)

$$\cos \delta_c = \left[\frac{\pi}{180}(141.7 - 38.3) + 1.615 \cos 141.7\right] \div 1.615 = 0.333$$

$\therefore$ δ_c = **70.54°**

For sustained fault

t *sec*	P_m *pu*	$P_e = P_m \sin \delta$ *pu*	$P_a = 1 - P_e$ *pu*	$1.41\ P_a$	$\Delta\delta$	δ *deg*
0–	1.615	1.0	0	0	–	38.3
0+	0	0	1.0	1.41	–	38.3
0_{av}	–	–	0.5	0.705	0.705	38.3
0.025	0	0	1	1.41	2.115	39.005
0.05	0	0	1	1.41	3.525	41.120
0.075	0	0	1	1.41	4.935	44.645
1.0	0	0	1	1.41	6.345	49.58
1.025	0	0	1	1.41	7.755	55.925
1.05	0	0	1	1.41	9.165	63.68
1.075	0	0	1	1.41	10.575	72.845
1.1						83.42

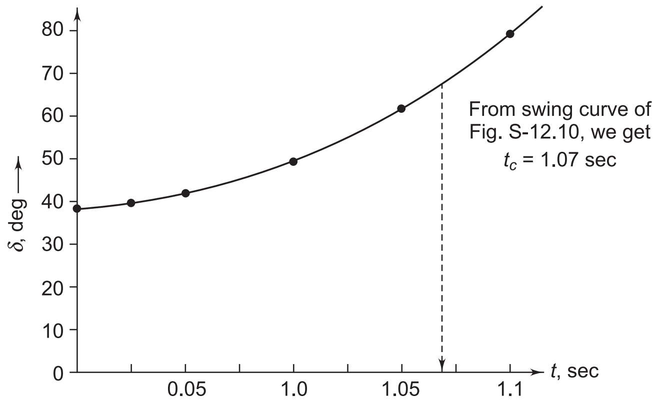


Fig. S-12.10 Swing curve for Prob 12.10 for sustained fault

12.11

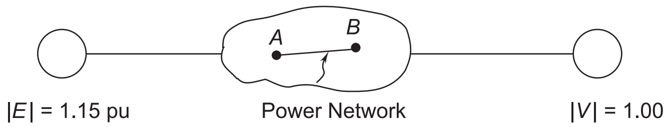


Fig. S-12.11 (a)

$$P_{eI}\text{ (prefault)} = \frac{1.15 \times 1}{0.5}\sin\delta = 2.3\sin\delta$$

$$P_{eII}\text{ (during fault)} = \frac{1.15 \times 1}{3}\sin\delta = 0.383\sin\delta$$

$$P_{eII'} = \frac{1.15 \times 1}{6}\sin\delta = 0.192\sin\delta$$

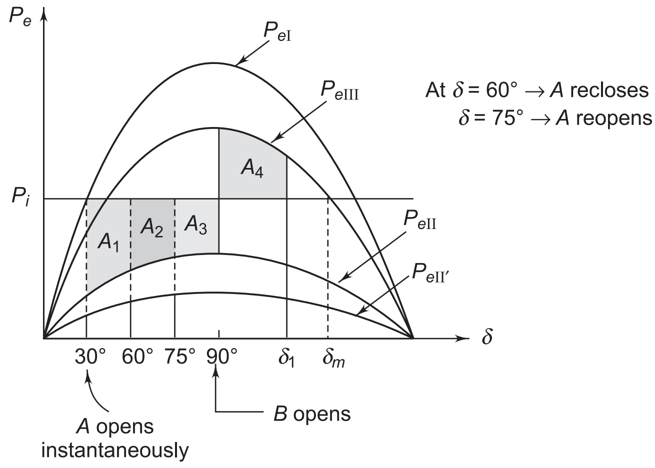


Fig. s-12.11 (b)

$$P_{eIII}\ (B\text{ opens}) = \frac{1.15}{0.6}\sin\delta = 1.92\sin\delta$$

$$P_i = 2.3 \sin 30° = 1.15,$$

$$\delta_m = 180° - \sin^{-1} \frac{1.15}{1.92} = 143.2°$$

For the system to be stable $\delta_1 < \delta_m$ and $A_1 + A_2 + A_3 = A_4$

$$A_1 = \int_{30°}^{60°} (P_i - 0.383 \sin \delta)\, d\delta = 1.15 \frac{\pi}{180} \times 30° + 0.383 (\cos 60° - \cos 30°) = 0.462$$

$$A_2 = \int_{60°}^{75°} (-0.192 \sin \delta + P_i)\, d\delta = 0.25$$

$$A_3 = \int_{75°}^{90°} (P_i - 0.383 \sin \delta)\, d\delta = 0.202$$

$$A_4 = \int_{90°}^{\delta_1} (1.92 \sin \delta - P_i)\, d\delta = -1.92 \cos \delta_1 - 0.02\, \delta_1 + 1.806$$

$$A_1 + A_2 + A_3 = A_4$$

$$0.462 + 0.202 + 0.250 = -1.92 \cos \delta_1 - 0.02\, \delta_1 + 1.806$$

$$1.92 \cos \delta_1 + 0.02\, \delta_1 - 0.892 = 0$$

By solving this equation, we can not obtain
$\delta_1 < \delta_m$ hence the system is *Unstable*. Alternatively, if the axes are plotted on a graph paper, it can be immediately seen that

$$A_1 + A_2 + A_3 > A_4$$

hence the system is *unstable*.

12.12

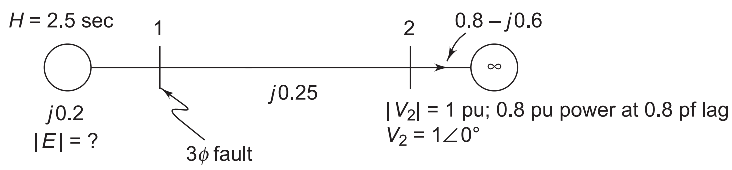


Fig. S-12.12

$$M = \frac{GH}{180 f} = \frac{1 \times 2.5}{180 \times 50} = 2.78 \times 10^{-4} \text{ sec}^2\text{/elect deg.}$$

$$I = \frac{0.8}{0.8 \times 1} = 1 \angle -36.9° = 0.8 - j\,0.6$$

$$E = 1 + j\,0.45\,(0.8 - j\,0.6) = 1.32 \angle 15.83°$$

I Prefault $P_{eI} = \frac{1.32 \times 1}{0.45} \sin \delta = 2.93 \sin \delta \;\therefore\; \delta_0 \;\therefore = \sin^{-1} \frac{0.8}{2.93} = 15.83°$

II During fault $P_{eII} = 0$ Choose $\Delta t = 0.05$ sec

III Post fault $P_{eIII} = P_{eI}$; $\quad \frac{(\Delta t)^2}{M} = \frac{(0.05)^2}{2.78 \times 10^{-4}} = 9.00$

t *sec*	P_m	$P_e = P_m \sin \delta$	$Pa = 0.8 - Pe$	$9\ Pa$	$\Delta\delta$ *day*	δ *deg*
0–	2.93	0.8	0	0	0	15.83°
0+	0	0	0.8	7.2		15.83°
0_{av}				3.6	3.6	15.83°
0.05	0	0	0.8	7.2	10.8	19.43
0.10	0	0	0.8	7.2	18.0	30.23
0.15–	0	0	0.8	7.2		48.23
0.15+	2.93	2.216	– 1.416	– 12.744		48.23
0.15_{av}			– 0.308	– 2.772	15.228	48.23
0.2	2.93	2.651	– 1.851	– 16.659	– 1.431	63.458
0.25	2.93					63.362
						62.027

Torque angle δ = 62.0270 at 250 milisecs.

12.13 50 Hz, 500 MVA, $|E|$ = 450 kV, $|V|$ = 400 kV, Δt = 0.05 sec, t_1 = 0.15 sec, H = 2.5 MJ/MVA, Load = 460 MW, X_I = 0.5 pu, X_{II} = 1.0 pu, X_{III} = 0.75 pu; $M = (1 \times 2.5)/(180 \times 50) = 2.778 \times 10^{-4}$ sec^2/elect. deg.

Base MVA = 500

Base kV = 400

G = 1pu

$$P_{eI} = \frac{1 \times 1.125}{0.5} \sin \delta = 2.25 \sin \delta;$$

$$\text{Prefault power transfer} = \frac{460}{500} = 0.92 \text{ pu}$$

$2.25 \sin \delta_0 = 0.92 \ \therefore \ \delta_0 = 24.14°$

During fault: $$P_{eII} = \frac{1 \times 1.125}{1} \sin \delta = 1.125 \sin \delta$$

Post fault $$P_{eIII} = \frac{1 \times 1.125}{0.75} \sin \delta = 1.5 \sin \delta;$$

$$\frac{(\Delta t)^2}{M} = \frac{(0.05)^2}{2.778 \times 10^{-4}} = 9$$

t *sec*	P_m	$P_e = P_m \sin \delta$	$P_a = 0.92 - P_e$	$9P_a$	$\Delta\delta$ *deg*	δ *deg*
0–	2.25	0.92	0	0	0	24.14
0+	1.125	0.46	0.46	4.14		24.14
0_{av}				2.07	2.07	24.14
0.05	1.125	0.496	0.424	3.816	5.886	26.21
0.1	1.125	0.597	0.323	2.907	8.793	32.1
0.15–	1.125	0.737	0.183	1.647		40.9

(Contd.)

t sec	P_m	$P_e = P_m \sin\delta$	$P_a = 0.92 - P_e$	$9P_a$	$\Delta\delta$ deg	δ deg
0.15+	1.5	0.982	– 0.062	– 0.56		40.9
0.15_{av}	1.5			0.543	9.336	40.9
0.2	1.5	1.15	– 0.23	– 2.07	7.27	50.24
0.25	1.5	1.265	– 0.345	– 3.105	4.16	57.50
0.3	1.5	1.32	– 0.4	– 3.6	0.56	61.66
0.35	1.5	1.327	– 0.407	– 3.663	– 3.1	62.22
0.4	1.5	1.287	– 0.357	– 3.213	– 6.313	59.12
0.45	1.5	1.194	– 0.274	– 2.466	– 8.779	52.81
0.5	1.5					44.04

System is STABLE

12.14 From Example 12.8

I Prefault $P_{eI} = 2 \sin\delta \therefore \delta_0 = \sin^{-1} 1/2 = 30°$

II During fault $P_{eII} = 0.5 \sin\delta$

III Post fault (after opening of circuit breakers)

$$P_{eIII} = 1.5 \sin\delta;\ M = \frac{1 \times 3.5}{180 \times 50} = 3.89 \times 10^{-4} \text{ sec}^2/\text{elec deg}$$

$$\text{Time to circuit breaker opening (3 cycles)} = \frac{3}{50} = 0.06 \text{ sec}$$

$$\text{Time to circuit breaker opening (8 cycles)} = \frac{8}{50} = 0.16 \text{ sec}$$

$$\Delta t = 0.05 \text{ sec};\ (\Delta t)^2/M = (0.05)^2 \big/ (3.89 \times 10^{-4}) = 6.426$$

Fault clears at 3 cycles

t	P_m	$P_e = P_m \sin\delta$	$P_a = 1 - P_e$	$6.426\ P_a$	$\Delta\delta$	δ_{deg}
0–	2.0	1.0	0.0			30.0
0+	0.5	0.25	0.75			30.0
0_{av}			0.375	2.41	2.41	30.0
0.05	0.5	0.268	0.732	4.70	7.11	32.41
0.10	1.5	0.954	0.046	0.296	7.406	39.520
0.15	1.5	1.095	– 0.095	– 0.61	6.796	46.926
0.20	1.5	1.209	– 0.209	– 1.343	5.453	53.722
0.25	1.5	1.288	– 0.288	– 1.850	3.603	59.175
0.30	1.5	1.333	– 0.333	– 2.139	1.464	62.778
0.35	1.5	1.350	– 0.350	– 2.249	– 0.785	64.242
0.40	1.5	1.341	– 0.341	– 2.191	– 2.976	63.451
0.45	1.5	1.305	– 0.305	– 1.959	– 4.935	60.481
0.50	1.5	1.236	– 0.236	– 1.516	– 6.451	55.546
0.55	1.5	1.133	– 0.133	– 0.854	– 7.305	49.095
0.60	1.5	1.0	0	0	– 7.305	41.79
						34.485

∴ System is STABLE.

Fault clears at 8 cycles

t_{sec}	P_m	$P_e = P_m \sin \delta$	$1 - P_e$	$6.426\ P_a$	$\Delta\delta$	δ deg
0–	2	1	0			30
0+	0.5	0.25	0.75			
0_{av}			0.375	2.41	2.41	30
0.05	0.5	0.268	0.732	4.7	7.11	32.41
0.10	0.5	0.318	0.682	4.383	11.493	39.52
0.15	0.5	0.389	0.611	3.926	15.419	51.013
→						
0.2	1.5	1.374	– 0.374	– 2.403	13.016	66.432
0.25	1.5	1.474	– 0.474	– 3.045	9.971	79.448
0.30	1.5	1.5	– 0.5	– 3.213	6.758	89.41
0.35	1.5	1.491	– 0.491	– 3.155	3.603	96.177
0.4	1.5	1.478	– 0.478	– 3.072	0.531	99.78
0.45	1.5	1.475	– 0.475	– 3.052	– 2.521	100.311
0.5	1.5	1.486	– 0.486	– 3.123	– 5.644	97.79
0.55	1.5	1.5	– 0.5	– 3.623	– 9.267	92.146
0.6	1.5	1.488	– 0.488	– 3.136	– 12.403	82.879
0.65						70.476

∴ System is STABLE.

SUSTAINED FAULT

0–	2.0	1.0	0.0			30.0
0+	0.5	0.25	0.75			
0_{av}			0.3750	2.41	2.41	30.0
0.05	0.5	0.268	0.732	4.7	7.11	32.41
0.1	0.5	0.318	0.682	4.383	11.493	39.52
0.15	0.5	0.389	0.611	3.926	15.419	51.013
0.2	0.5	0.458	0.542	3.482	18.901	66.432
0.25	0.5	0.498	0.502	3.226	22.127	85.333
0.3	0.5	0.477	0.523	3.361	25.488	107.46
0.35	0.5	0.37	0.63	4.05	29.538	132.948
0.4	0.5	0.15	0.85	5.462	35.0	162.486
0.45	0.5	– 0.1502	1.1502	7.391	42.391	197.486
0.5	0.5					239.877

12.15

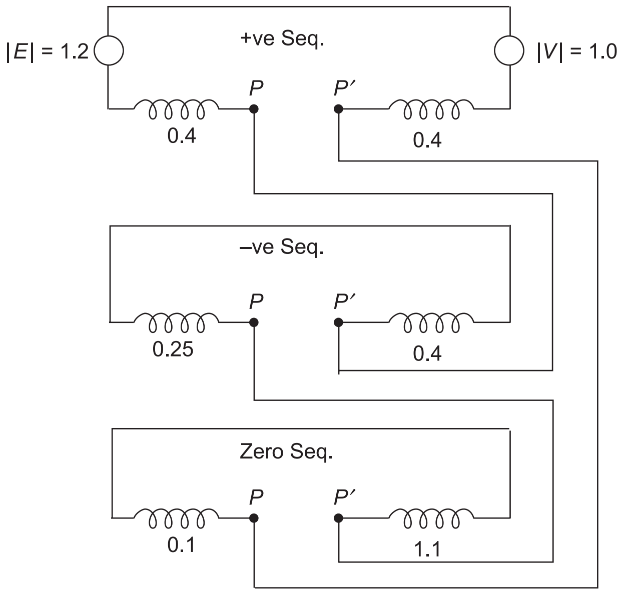


Fig. S-12.15 (a) Connection of sequence networks with both faulted lines switched off

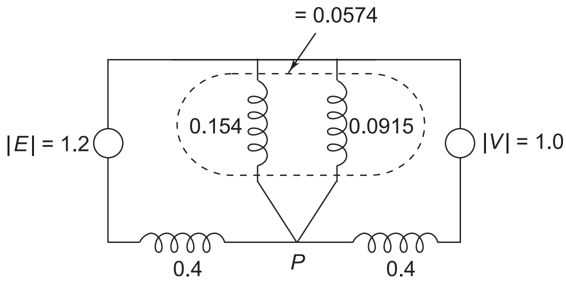


Fig. S-12.15 (b) Connection of sequence networks for LIG fault as P

Transfer reactances are:

$$X_{12} \text{ (2 LG fault)} = 0.4 + 0.4 + \frac{0.4 \times 0.4}{0.0574} = 3.59$$

$$X_{12} \text{ (both faulted lines open)} = 0.8 + 0.65 + 1.2 = 2.65$$

$$X_{12} \text{ (line healthy)} = 0.8$$

$$\textit{Prefault } P_{eI} = \frac{1.2 \times 1}{0.8} \sin \delta = 1.5 \sin \delta$$

$\therefore \qquad \delta_0 = \sin^{-1} \frac{1}{1.5} = 41.8°$

During fault $P_{eII} = \frac{1.2 \times 1}{3.59} \sin \delta = 0.334 \sin \delta$

During three-pole switching $P_{eIII} = 0$

During single pole switching

$$P_{eIII} = \frac{1.2 \times 1}{2.65} \sin \delta = 0.453 \sin \delta$$

Post fault $P_{eIV} = P_{eI} = 1.5 \sin \delta$

Swing curve calculations:

$$H = 4.167 \text{ MJ/MVA},$$

$$M = \frac{4 \times 167}{180 \times 50} = 4.63 \times 10^{-4} \text{ sec}^2\text{/elect. deg}$$

Taking $\Delta t = 0.05$ sec

$$\frac{(\Delta t)^2}{M} = (0.05)^2 / (4.65 \times 10^{-4}) = 5.4$$

Time when single/three pole switching occurs = 0.075 sec (during middle of Δt)

Time when reclosing occurs = 0.325 (during middle of Δt).

(i) Swing curve calculations—*three-pole switching*

t *sec.*	P_m	P_e	$P_a = 1 - P_e$	$5.4P_a$	$\Delta\delta$	δ_{deg}
0–	1.500	1.0	0	0		41.8
0+	0.334	0.223	0.777	4.198		41.8
0_{av}				2.099	2.1	41.8
0.05 →	0.334	0.232	0.768	4.147	6.25	43.9
0.10	0	0	1.0	5.4	11.65	50.15
0.15	0	0	1.0	5.4	17.05	61.80
0.20	0	0	1.0	5.4	22.45	78.85
0.25	0	0	1.0	5.4	27.85	101.30
0.30 →	0	0	1.0	5.4	33.25	129.15
0.35	1.5	0.454	0.546	2.951	36.201	162.4
0.4	1.5	– 0.478	1.478	7.981	44.18	198.60
0.45	1.5	– 1.333	2.333	12.598	56.778	242.78
0.5						299.56

The system is OBVIOUSLY UNSTABLE

(ii) Swing curve calculations—*single-pole switching*

0–	1.5	1.0	0	0		41.8
0+	0.334	0.223	0.777	4.198		41.8
0_{av}				2.099	2.1	41.8
0.05	0.334	0.232	0.768	4.147	6.25	43.9
→						
0.10	0.453	0.348	0.652	3.522	9.776	50.15
0.15	0.453	0.392	0.608	3.283	13.05	59.92
0.20	0.453	0.433	0.567	3.061	16.11	72.97
0.25	0.453	0.453	0.547	2.954	19.06	89.08
0.30	0.453	0.430	0.57	3.08	22.14	108.14
→						
0.35	1.50	1.144	– 0.144	– 0.8	21.34	130.28
0.4	1.50	0.713	0.287	1.550	22.89	151.62
0.45	1.50	0.1435	0.856	4.622	27.51	174.51
0.5	1.50	– 0.562	1.562	8.43	35.94	202.02
0.55	1.50	– 1.271	2.271	12.26	48.20	237.96
						286.16

The System is UNSTABLE

Note: In this problem in case of LLG fault in spite of single pole switching the system is found to be UNSTABLE.